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A Manual of
PROBLEMS IN STATISTICS

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PREFACE

This book was written as a companion volume to *Statistical Methods* by Frederick C. Mills. However, the *Manual* may be used in connection with other modern statistics texts, since the selection of problem material has been quite general.

In the main, an attempt has been made not only to provide problem material covering all phases of current statistical technique but also to permit the instructor considerable choice of material for class assignments. The inclusion of several problems dealing with each division of the subject also makes possible separate assignments for individual groups within a class if such treatment is desired.

In order that students might have between the covers of a single book not only the data of their problems but also the chief statistical tables needed in the solution of these problems, permission has been given by Dr. Mills and his publishers (Henry Holt & Co.) to include the complete appendix of statistical tables included in his recently revised edition of *Statistical Methods*. Acknowledgment is also made for permission by Dr. R. A. Fisher and his publishers (Oliver and Boyd, Edinburgh) to reprint excerpts from various tables from his *Statistical Methods for Research Workers*. Professor Snedecor has extended the courtesy of permitting the inclusion of his table of "5% and 1% points for the distribution of F" given in his *Statistical Methods*, 1938 (Collegiate Press, Inc., Ames, Iowa).

Two appendices are included in the manual. Appendix A contains not only material cited in the various problems but also data that may be used for supplementary assignments. Appendix B is made up of the prepared statistical tables mentioned above.

S. D.

August 4, 1941
Columbia University

A. INTRODUCTORY EXERCISES

The exercises of this section are of an elementary nature designed to refresh the student's mind in the elementary principles of algebra which he will be called upon to use in statistical work.

1. Simplify the following expressions:

a. $(3x-4y)+(2x+3y)$
b. $(-2a+3b)+(4a-6b)$
c. $(12.5h-3.7m)+(-4.8h+9.6m)$
d. $(12x-8y)-(6x+4y)$
e. $(-9a-3y)-(12a+7y)$
f. $(-6r+9.2s)-(7r-8.8s)$
g. $5d(3d^2-4h)$
h. $3a(2b-6c)$
i. $(2a-3b)(4a+6b)$
j. $(3r-5s)(-3r+5s)$
k. $(4h^2+9k)(-3h-4k^3)$
l. $(a^3+3a^2b+3ab^2+b^3)\div(a+b)$
m. $(6a^2+5ab-6b^2)\div(3a-2b)$
n. $(-6a^3-6a^2b-8ab-8b^2)$
$\div(-2a-2b)$
o. $(a+b)^4$
p. $(h+t)^6$
q. $(r^2-2s)^5$
r. $50(5)^{-2}$
s. $(4)^0(16)^{1/2}(-2)$

2. Solve the following simultaneous equations:

a. $2x-4y=-8$
$5x+3y=19$

b. $2.5a-3.2b=-6$
$-1.1a+5.4b=22.6$

c. $3.2r-5.1s=-35$
$-1.8r+1.7s=8$

d. $6x-4y+3z=33$
$5x+3y-7z=18$
$8x-6y+5z=45$

e. $2.4a+3.6b-2.2c=2.80$
$-3.0a-4.2b+4.4c=6.76$
$1.8a+4.8b-8.8c=-30.8$

f. $1.5r-2.2s+4.6t=20.45$
$-.8r+1.5s-1.8t=-9.6$
$3.2r-.8s+2.5t=13.9$

g. $3a-4b+6c-7d=-27$
$4a-b+5c+2d=29$
$6a+3b-3c-3d=0$
$a+5b-4c+6d=40$

h. $3.4r-3.6s+4.5t-7.0u=43.5$
$1.6r+1.6s-6.3t+9.0u=-76.5$
$-2.5r-3.5s+7.2t-3.1u=49.5$
$.7r-.8s-1.6t+12.0u=-71.0$

3. By the use of logarithms perform the following operations:

a. (125)(42)
b. (320)(16)
c. (34)(.345)
d. (.015)(.0592)
e. (1.342)(27.9)
f. (149.02)(.00473)
g. (135)(.042)
h. (4.83)(187.68)
i. (.9884)(.0396)
j. (.03495)(59.63)
k. (379.2)(5667)
l. (.06948)(25410)
m. $29070\div85$
n. $2835\div450$
o. $1881\div34.2$
p. $47.06\div.724$
q. $1364\div.0864$
r. $.7485\div137.3$
s. $.6086\div.0664$
t. $.01452\div2.75$
u. $52.11\div1.437$
v. $4.771\div1900$
w. $5.06\div84.7$

4. By the use of logarithms perform the following operations:

a. $(40)^3$
b. $(.02)^4$
c. $(5)^5$
d. $(.004)^5$
e. $(43)^7$
f. $(16)^9$
g. $(39.2)^{1/2}$
h. $(1842)^6$

i. $(.0625)^{1/4}$
j. $(.04729)^{-1}$
k. $(8.275)^{-1/2}$
l. $\sqrt[3]{1728}$
m. $\sqrt[3]{1034}$
n. $\sqrt[4]{176400}$
o. $\sqrt[10]{129.6}$
p. $\sqrt[7]{359}$
q. $(154)^{2/3}$
r. $(4204)^{3/7}$
s. $(145.6)(35.4) \div (13.5)(76.23)$
t. $(134.6)^{1/2}(187)^{4} \div (18.5)(64)^{-1/4}$

5. Solve the following expressions for the required term:

a. $ax^2 - 2ay - 3az^2 = 12$, for a.
b. $(\Sigma Y) = na + b(\Sigma X)$, for a.
c. $(\Sigma X^2 Y) = a(\Sigma X^2) + b(\Sigma X^3) + c(\Sigma X^4)$, for c.
d. $r^2 = 1 - \frac{(S_y^2)}{(\sigma_y^2)}$, for S_y.
e. $\sigma_r = \frac{r}{\sqrt{N-1}}$, for N.
f. $r = \frac{\Sigma xy}{N\sigma_x\sigma_y}$, for σ_x.

B. GRAPHIC PRESENTATION

1. Present the following equations graphically:

a. $y=a+bx$

(1) when $a=5$ and $b=2$
(2) when $a=5$ and $b=-2$
(3) when $a=-5$ and $b=2$
(4) when $a=-5$ and $b=-2$

b. $y=a+bx$

(1) when $a=3$ and $b=3$
(2) when $a=3$ and $b=-3$
(3) when $a=-3$ and $b=3$
(4) when $a=-3$ and $b=-3$

c. $y=a+bx$

(1) when $a=4$ and $b=1.5$
(2) when $a=4$ and $b=-1.5$
(3) when $a=-4$ and $b=1.5$
(4) when $a=-4$ and $b=-1.5$

d. $y=a+bx+cx^2$

(1) when $a=5$, $b=2$ and $c=.5$
(2) when $a=5$, $b=2$ and $c=-.5$
(3) when $a=5$, $b=-2$ and $c=.5$
(4) when $a=5$, $b=-2$ and $c=-.5$

e. $y=a+bx+cx^2$

(1) when $a=4$, $b=2$ and $c=.4$
(2) when $a=4$, $b=2$ and $c=-.4$
(3) when $a=4$, $b=-2$ and $c=.4$
(4) when $a=4$, $b=-2$ and $c=-.4$

f. $y=a+bx+cx^2$

(1) when $a=-6$, $b=3$ and $c=.6$
(2) when $a=-6$, $b=3$ and $c=-.6$
(3) when $a=-6$, $b=-3$ and $c=.6$
(4) when $a=-6$, $b=-3$ and $c=-.6$

g. $y=a+bx+cx^2+dx^3$

(1) when $a=1.4$, $b=-1.5$, $c=-.2$ and $d=.2$
(2) when $a=1.4$, $b=-1.5$, $c=.2$ and $d=-.2$
(3) when $a=1.4$, $b=-1.5$, $c=-.2$ and $d=-.2$
(4) when $a=1.4$, $b=-1.5$, $c=.2$ and $d=.2$
(5) when $a=1.4$, $b=1.5$, $c=-.2$ and $d=.2$
(6) when $a=1.4$, $b=1.5$, $c=.2$ and $d=-.2$
(7) when $a=1.4$, $b=1.5$, $c=-.2$ and $d=-.2$
(8) when $a=1.4$, $b=1.5$, $c=.2$ and $d=.2$

h. $y=\frac{1}{a+bx}$

(1) when $a=0$ and $b=2$
(2) when $a=2$ and $b=2$
(3) when $a=1$ and $b=3$
(4) when $a=1$ and $b=-3$

i. $y=\frac{1}{x}$

j. $y=k+ab^x$

(1) when $k=10$, $a=2$ and $b=2$
(2) when $k=10$, $a=2$ and $b=1.2$
(3) when $k=20$, $a=4$ and $b=1.1$
(4) when $k=20$, $a=4$ and $b=1.2$

k. $\frac{1}{y}=a+bx$

(1) when $a=2$ and $b=4$
(2) when $a=2$ and $b=-4$
(3) when $a=4$ and $b=1.5$
(4) when $a=4$ and $b=-1.5$

l. $y=ab^{c^x}$

(1) when $a=50$, $b=.025$ and $c=.9$
(2) when $a=60$, $b=.035$ and $c=.9$
(3) when $a=1000$, $b=.02$ and $c=.8$
(4) when $a=2000$, $b=.03$ and $c=.8$

m. $\frac{1}{y}=a+bc^x$

(1) when $a=.10$, $b=.2$ and $c=.6$
(2) when $a=.10$, $b=.2$ and $c=.5$
(3) when $a=.20$, $b=.3$ and $c=.5$
(4) when $a=.20$, $b=.3$ and $c=.6$

2. Present the following equations graphically on semilogarithmic paper (vertical scale logarithmic) and on arithmetic paper:

a. $y=a^x$

(1) when $a=2$
(2) when $a=4$
(3) when $a=.2$
(4) when $a=.4$

b. $y=ab^x$

(1) when $a=100$ and $b=1.10$
(2) when $a=100$ and $b=.90$
(3) when $a=200$ and $b=1.40$
(4) when $a=200$ and $b=.60$

3. Present the following equations graphically on double logarithmic paper and on arithmetic paper:

a. $y=x^b$

(1) when $b=2$
(2) when $b=3$
(3) when $b=1.2$

b. $y=ax^b$

(1) when $a=2$ and $b=2$
(2) when $a=.5$ and $b=3$
(3) when $a=10$ and $b=1.5$
(4) when $a=20$ and $b=2.5$

4. On arithmetic cross section paper plot the data relating to annual rubber imports into the United States given in the table below. The figures plotted are to be presented in a table accompanying the chart.

Year	Total Annual Rubber Imports into United States in thousand long tons	Average Monthly World Sales of General Motors Corporation (U. S. and Canadian plants), in thousands of cars
1921	185.4	17.9
1922	301.1	38.1
1923	309.1	66.5
1924	329.4	48.9
1925	400.4	69.7
1926	417.6	102.9
1927	431.2	130.2
1928	439.7	150.9
1929	565.1	158.3
1930	487.6	96.5
1931	501.8	86.1
1932	414.7	43.8
1933	418.9	66.8
1934	463.0	94.0
1935	467.1	130.4
1936	488.1	155.5
1937	600.5	160.7
1938	412.1	92.3
1939	499.3	128.6
1940	817.9	168.8

Sources: U. S. Bureau of Foreign and Domestic Commerce; General Motors Corporation

5. Using the data in the above table showing the average monthly world sales of the General Motors Corporation, construct an arithmetic graph and a table of the data to accompany the chart.

6. Using data from Table 1 or Table 2 of Appendix A as assigned by the instructor, construct an arithmetic chart with an accompanying table of the series plotted.

7. The following table giving the monthly sales of Sears, Roebuck & Company for the years 1936–1940 is to be presented in graphic form on an arithmetic chart. The graph is to be accompanied by an appropriate table of the data.

Monthly Sales of Sears, Roebuck & Co., 1936–1940 (thousands of dollars)

	1936	1937	1938	1939	1940
Jan.	27,672	31,849	30,620	33,551	40,548
Feb.	27,580	31,671	30,449	34,901	40,836
Mar.	36,081	43,694	41,071	49,768	50,899
Apr.	39,011	49,585	44,857	51,236	56,372
May	44,923	53,487	43,463	59,613	65,978
June	46,359	52,198	43,820	56,768	63,313
July	39,634	43,216	36,316	43,941	51,352
Aug.	38,903	41,575	39,934	48,259	58,820
Sept.	47,617	52,781	49,167	62,751	65,650
Oct.	58,135	58,626	53,345	67,246	76,920
Nov.	49,014	50,262	51,215	60,330	73,324
Dec.	66,433	64,872	68,622	82,427	95,873

Source: Survey of Current Business

8. Plot the following data on electric power production on one-cycle semi-logarithmic paper. The graph should be accompanied by an appropriate table of the data plotted.

Annual Production of Electric Power in the United States, by Source, for the years 1921–1940 (billion kilowatt hours)

Year	Fuel	Water power
1921	26.0	14.9
1922	30.4	17.1
1923	36.3	19.2
1924	39.0	19.8
1925	43.5	22.2
1926	47.6	26.1
1927	50.3	29.0
1928	53.2	33.4
1929	62.7	33.2
1930	62.9	31.7
1931	61.2	29.6
1932	49.1	33.2
1933	50.7	34.1
1934	57.1	33.7
1935	59.4	39.0
1936	72.7	39.5
1937	77.4	44.5
1938	71.8	44.8
1939	86.3	44.0
1940	97.2	47.7

Source: **U. S. Federal Power Commission**

9. Plot the following data on United States imports and exports on one-cycle semilogarithmic paper, including with the graph a table of the data.

Total Value of Imports and Exports of the United States, for the years 1921–1940
(millions of dollars)

Year	Exports (including re-exports)	General imports
1921	4,485	2,509
1922	3,832	3,113
1923	4,167	3,792
1924	4,591	3,610
1925	4,910	4,227
1926	4,809	4,431
1927	4,865	4,185
1928	5,128	4,091
1929	5,241	4,399
1930	3,843	3,061
1931	2,424	2,091
1932	1,611	1,323
1933	1,675	1,450
1934	2,133	1,655
1935	2,283	2,047
1936	2,456	2,423
1937	3,349	3,084
1938	3,094	1,960
1939	3,178	2,318
1940	4,023	2,624

Source: U. S. Bureau of Foreign and Domestic Commerce

10. The following data showing the volume of United States foreign trade cleared in foreign and United States vessels are to be plotted on one-cycle semilogarithmic paper. Different colored lines or coded ink lines should be used to distinguish the individual series. A table of the three series should accompany the graph.

Ship Clearances in United States Foreign Trade, by Registry, 1930–1940
(millions of net tons)

Year	Foreign vessels	United States vessels	Total
1930	49.75	31.56	81.31
1931	46.64	26.86	73.50
1932	40.58	23.87	64.45
1933	38.86	22.43	61.29
1934	40.91	22.80	63.71
1935	42.76	22.13	64.89
1936	46.00	20.06	66.06
1937	52.94	19.94	72.88
1938	52.12	18.73	70.85
1939	52.14	18.12	70.26
1940	42.18	20.36	62.54

Source: U. S. Bureau of Foreign and Domestic Commerce and U. S. Bureau of Customs

11. By inspection of the chart prepared for problem B10, determine for each series the year of greatest rate of increase over the preceding year.

12. Using two-cycle semilogarithmic paper, plot the following data on Canadian bond issues. By inspection, determine which year shows the greatest percent of decrease over the preceding year.

Total New Bond Issues in Canada, 1930–1940 (millions of dollars)

Year	Amount issued
1930	767.2
1931	1,250.8
1932	473.1
1933	569.6
1934	638.0
1935	1,016.5
1936	1,299.1
1937	1,239.8
1938	1,132.9
1939	1,439.1
1940	2,300.5

Source: Survey of Current Business

13. The following table giving the value of life insurance written annually in the United States for the years 1921–1940 is to be presented on a graph of two-cycle semilogarithmic paper. Use contrasting lines to distinguish the separate series.

Total Value of Life Insurance Written, 1921–1940 (representing 40 companies having about 82 per cent of all United States business)

	in millions of dollars		
Year	Group	Industrial	Ordinary
1921	111.1	1,257.8	4,462.9
1922	276.4	1,418.8	4,913.5
1923	520.0	1,720.1	5,879.5
1924	597.8	1,963.6	6,256.0
1925	998.8	2,359.2	7,253.2
1926	1,050.6	2,566.1	7,667.6
1927	836.4	2,667.3	7,678.0
1928	1,336.3	2,692.5	8,083.9
1929	1,185.4	2,898.2	8,684.1
1930	1,092.2	2,851.1	8,397.0
1931	796.2	2,797.2	7,409.8
1932	627.1	2,477.3	5,931.3
1933	357.2	2,320.9	5,085.8
1934	497.0	2,527.2	5,528.9
1935	665.1	2,521.3	5,581.0
1936	584.8	2,695.6	5,371.2

1937	760.9	2,640.1	5,591.8
1938	476.4	2,178.9	4,867.4
1939	803.1	1,484.1	5,015.7
1940	679.3	1,581.9	5,056.6

Source: Association of Life Insurance Presidents

14. (a) Plot the following four series on a sheet of two-cycle semilogarithmic paper using coded lines to distinguish between the individual series. A table of the data plotted should accompany the graph.

(b) Prepare scales of increase, decrease and comparison and estimate, for each of the following series

(1) the maximum per cent of increase of one year over a preceding year.
(2) the maximum per cent of decrease of one year over a preceding year.
(3) the per cent the largest figure of each type of revenue is of the total revenue for the corresponding year.

Annual Operating Revenue of Class I Steam Railways in the United States, by Source, 1930–1940 (millions of dollars)

Year	Freight revenue	Passenger revenue	Other revenue	Total revenue
1930	4,083.3	729.6	468.5	5,281.4
1931	3,254.8	551.0	382.5	4,188.3
1932	2,449.8	377.0	298.6	3,125.4
1933	2,492.8	329.3	273.4	3,095.5
1934	2,633.4	346.3	291.9	3,271.6
1935	2,796.3	357.9	296.1	3,450.3
1936	3,307.6	412.4	331.7	4,051.7
1937	3,378.2	442.8	345.2	4,166.2
1938	2,857.9	405.4	301.5	3,564.8
1939	3,251.1	416.9	327.1	3,995.1
1940	3,537.2	417.3	342.1	4,296.6

Source: Interstate Commerce Commission Reports

15. Plot the following data on three-cycle semilogarithmic paper. Note that the data are for selected years only. Include a table of the data plotted with the graph.

Average Monthly Cargo Tonnage Carried by the Federal Barge Line on the Mississippi Division, for Selected Years

Year	Av. mo. tonnage in thousand short tons
1919	8.7
1921	36.9
1923	59.3
1925	75.9

1927	104.3
1929	116.6
1931	104.2
1933	112.6
1935	128.9
1937	149.9
1939	157.9

Source: Survey of Current Business

16. The following data on bauxite imports into the United States are to be presented on three-cycle semilogarithmic paper with a table of the data plotted accompanying the graph. Note that the data are for selected years only.

Total Annual Bauxite Imports into the United States, for Selected Years

Year	Imports in thousand long tons
1916	2.2
1918	3.8
1920	49.0
1922	27.4
1924	202.0
1926	281.6
1928	350.1
1930	415.1
1932	205.6
1934	166.7
1936	322.8
1938	455.7
1940	629.6

Source: U. S. Bureau of Foreign and Domestic Commerce

17. Plot the data of the table below on a sheet of three-cycle semilogarithmic paper. Include with the graph a table of the original data plotted. Prepare a scale of increase and estimate the greatest per cent of increase shown between any two years for each series.

Total Miles Flown and Total Passenger Miles Flown by Scheduled United States Airlines for the Years 1930–1940

Year	Total miles flown (millions of miles)	Total passenger miles flown (millions of miles)
1930	32.0	84.0
1931	42.8	106.4
1932	45.6	127.0

1933	48.8	173.5
1934	50.5	187.9
1935	55.4	313.9
1936	63.8	435.7
1937	66.1	476.6
1938	69.7	557.7
1939	82.6	749.8
1940	108.8	1,147.4

Source: U. S. Civil Aeronautics Authority

18. Construct a vertical bar chart from the following data on United States fire losses.

Total Fire Losses in the United States, 1930–1940 (millions of dollars)

Year	Estimated fire loss
1930	502.0
1931	451.6
1932	400.9
1933	271.5
1934	271.2
1935	235.3
1936	266.7
1937	255.0
1938	265.6
1939	313.5
1940	306.5

Source: Survey of Current Business

19. Construct a horizontal bar chart from the following data.

Liabilities Involved in Failures of Manufacturing Concerns in the United States for the year 1940, Classified According to Product Manufactured (millions of dollars)

Product manufactured	Total liabilities
Food and kindred products	14.0
Textile-mill products and apparel	11.0
Lumber and products	6.3
Paper, printing and publishing	5.7
Iron and steel and products	3.1
Machinery	3.0
Transportation equipment	2.8
Chemicals and allied products	2.7
Leather and leather products	2.0
Stone, clay and glass products	1.3

Source: Survey of Current Business

20. Show the change in the relative importance of the two series given in the table below by a compound bar chart (i.e., without space between the two bars for each year), using appropriate shading to distinguish between the two series.

Total Number of Emigrants Leaving and Immigrants Coming to the United States, 1930–1940

Year	Immigrants	Emigrants
1930	52,932	180,252
1931	89,568	43,356
1932	97,512	28,008
1933	51,696	23,904
1934	39,180	34,368
1935	36,492	34,908
1936	30,516	41,592
1937	24,744	62,616
1938	25,032	76,080
1939	26,352	73,560
1940	16,414	60,901

Source: U. S. Immigration and Naturalization Service

21. Construct a chart presenting the following data in a series of component-part bars.

Location of Cotton Stocks in the United States, 1929–1940
Monthly Averages in Thousands of Bales

Year	Total	Per cent	On farms and in transit	% of total	Ware-houses	% of total	Mills	% of total
1929	8,181	100	3,577	43.72	3,282	40.12	1,322	16.16
1930	9,776	100	3,447	35.26	4,970	50.84	1,359	13.90
1931	13,227	100	5,050	38.18	6,985	52.81	1,192	9.01
1932	14,878	100	4,940	33.20	8,589	57.73	1,349	9.07
1933	13,675	100	4,098	29.97	8,234	60.21	1,343	9.82
1934	11,906	100	2,850	23.94	7,745	65.05	1,311	11.01
1935	11,427	100	3,024	26.46	7,407	64.82	996	8.72
1936	10,459	100	2,881	—	6,350	—	1,228	—
1937	12,120	100	4,315	—	6,224	—	1,581	—
1938	16,231	100	2,843	—	11,896	—	1,492	—
1939	17,427	100	2,631	—	13,541	—	1,255	—
1940	16,321	100	3,391	—	11,618	—	1,312	—

Source: New York Cotton Exchange

22. Construct a chart presenting the data in problem 21 in a series of relative component-part bars.

23. Construct a chart presenting the following data in a series of component-part bars.

Total Ordinary Life Insurance Written in the United States, 1935–1940, by Geographical Regions, in Millions of Dollars

Year	Total	New England	Middle Atlantic	North Central	South Atlantic	South Central	Mountain and Pacific
1935	7,281	541	2,175	2,348	653	812	752
1936	7,344	546	2,153	2,371	657	818	799
1937	7,533	562	2,162	2,464	675	849	821
1938	6,527	455	1,846	2,062	599	830	735
1939	6,426	495	1,745	2,104	606	756	720
1940	6,564	493	1,789	2,148	651	758	725

Source: Life Insurance Sales Research Bureau

24. Construct a chart presenting the data in problem 23 in a series of relative component-part bars.

25. Construct a chart presenting the following data in a series of component-part bars.

Annual Production of Boots and Shoes in the United States, by Kind, 1930–1940 (millions of pairs)

Year	Boys' and youths'	Infants'	Misses' and children's	Men's	Women's	Total
1930	18.53	18.56	32.04	77.15	112.63	258.91
1931	20.05	18.54	34.31	77.42	112.61	262.93
1932	18.10	15.65	33.60	74.50	113.94	255.79
1933	19.94	18.58	33.18	88.82	130.74	291.26
1934	17.35	19.45	34.52	91.39	133.04	295.75
1935	17.84	21.17	37.27	99.53	145.24	321.05
1936	16.57	21.61	36.84	103.79	161.86	340.67
1937	17.48	22.79	39.91	102.90	149.68	332.76
1938	17.14	21.31	40.19	96.66	147.76	323.06
1939	16.86	24.08	43.99	103.75	167.70	356.38
1940	15.33	21.55	39.71	102.08	149.89	328.56

Source: U. S. Bureau of the Census

26. Construct a chart presenting the data in problem 25 in a series of relative component-part bars.

27. Construct a separate pie diagram for each year of the following data. It is important that the three diagrams have the same diameter.

Construction Contracts Awarded in 37 States for Non-Residential Buildings, by Kinds, 1937–1939 (millions of dollars)

Kind of building	1937 Valuation	1937 % of total	1938 Valuation	1938 % of total	1939 Valuation	1939 % of total
Commercial	297	25.7	216	20.1	247	25.6
Education and Science	223	19.3	334	31.1	202	20.9
Manufacturing	313	27.0	121	11.3	175	18.1
Hospital and Institutional	82	7.1	116	10.8	83	8.6
Public	105	9.1	114	10.6	110	11.4
Religious and Miscellaneous	52	4.5	64	6.0	67	6.9
Social and Recreational	84	7.3	108	10.1	82	8.5
Total	1,156	100.0	1,073	100.0	966	100.0

Source: F. W. Dodge Corporation

28. Construct a separate pie diagram for each year of the following data. It is important that the six figures have the same diameter.

Total Value of United States Exports (including re-exports), by Grand Divisions for the years 1935–1940 (in millions of dollars)

Year	Total	Africa	Asia and Oceania	Europe	Northern North America	Southern North America	South America
1935	2,283	96	452	1,029	330	202	174
1936	2,456	114	478	1,043	392	225	204
1937	3,349	152	679	1,360	519	321	318
1938	3,094	118	610	1,326	476	264	300
1939	3,175	115	640	1,284	502	304	330
1940	4,023	161	712	1,645	729	341	435

Source: Bureau of Foreign and Domestic Commerce

29. Construct five pie diagrams from the following data, using a constant diameter.

Annual Production for Sale (outside the industry) of Steel Products in the United States, 1936–1940 (in thousand short tons)

Type of Product	1936	1937	1938	1939	1940
Sheets	7,748	8,337	4,944	8,505	10,181
Merchant bars	3,750	3,734	1,756	3,221	4,388
Pipe and tube	3,348	3,941	2,411	3,506	3,959

Wire and wire products	2,950	2,869	2,065	3,154	3,729
Strip	2,881	2,830	1,356	1,843	2,138
Plates	2,579	3,374	1,624	2,794	4,128
Heavy structural shapes	2,501	2,789	1,554	2,545	3,150
Tin plate	2,362	2,758	1,618	2,561	2,694
Rails	1,341	1,581	686	1,287	1,644
Miscellaneous	5,467	6,132	3,342	5,539	9,887
Total	34,927	38,345	21,356	34,955	45,898

Source: American Iron and Steel Institute

C. DESCRIPTION OF THE FREQUENCY DISTRIBUTION

1. The following is a tabulation of the average daily wages of 158 employees of a steel foundry in Aurora, Illinois during the month of June, 1941.

~~$8.75~~	~~$8.30~~	~~$8.20~~	~~$8.00~~	~~$8.00~~	~~$7.85~~	~~$7.90~~	~~$8.87~~	~~$8.62~~
~~7.80~~	~~8.40~~	~~7.00~~	~~8.20~~	~~8.05~~	~~8.15~~	~~8.09~~	~~8.90~~	~~7.90~~
~~8.00~~	~~8.60~~	~~8.15~~	~~8.30~~	~~8.30~~	~~8.00~~	~~8.20~~	~~8.70~~	~~8.60~~
~~8.15~~	~~7.86~~	~~8.40~~	~~7.85~~	~~7.70~~	~~8.12~~	~~8.55~~	~~8.22~~	~~8.95~~
~~8.15~~	~~8.48~~	~~7.87~~	~~8.15~~	~~8.12~~	~~8.12~~	~~8.05~~	~~8.58~~	~~8.72~~
~~8.12~~	~~8.05~~	~~8.35~~	~~8.48~~	~~8.32~~	~~8.20~~	~~7.87~~	~~7.53~~	~~8.00~~
~~7.55~~	~~7.87~~	~~8.48~~	~~7.63~~	~~7.90~~	~~7.73~~	~~8.15~~	~~8.00~~	~~8.20~~
~~8.05~~	~~8.37~~	~~8.90~~	~~8.05~~	~~8.15~~	~~8.35~~	~~8.35~~	~~8.22~~	~~8.09~~
~~8.25~~	~~8.00~~	~~7.41~~	~~8.25~~	~~7.86~~	~~7.44~~	~~7.92~~	~~8.35~~	~~7.73~~
~~7.85~~	~~8.09~~	~~8.37~~	~~8.45~~	~~8.09~~	~~8.00~~	~~8.05~~	~~8.12~~	~~8.35~~
~~8.32~~	~~7.90~~	~~7.92~~	~~8.45~~	~~8.09~~	~~8.15~~	~~8.15~~	~~8.48~~	~~7.92~~
~~8.09~~	~~8.12~~	~~8.15~~	~~8.75~~	~~7.83~~	~~8.22~~	~~8.32~~	~~7.61~~	~~8.22~~
~~8.22~~	~~8.32~~	~~7.73~~	~~8.37~~	~~8.35~~	~~7.82~~	~~8.45~~	~~8.05~~	~~7.86~~
~~8.22~~	~~8.30~~	~~8.52~~	~~8.09~~	~~7.92~~	~~8.05~~	~~7.90~~	~~7.85~~	~~8.15~~
~~7.49~~	~~7.70~~	~~7.85~~	~~8.09~~	~~8.84~~	~~8.25~~	~~8.14~~	~~8.15~~	~~8.25~~
~~8.05~~	~~8.00~~	~~8.25~~	~~8.54~~	~~8.70~~	~~9.02~~	~~8.09~~	~~7.70~~	~~7.87~~
~~8.15~~	~~8.14~~	~~7.57~~	~~7.80~~	~~8.45~~	~~8.60~~	~~8.30~~	~~8.09~~	~~8.25~~
~~8.25~~	~~7.99~~	~~8.32~~	~~7.70~~	~~8.09~~				

(a) Arrange these items in the form of an array, listing the items in order of size from smallest to largest.

(b) Construct a frequency distribution from the array, using a class-interval of $.15 with $7.475 as the midpoint of the first class-interval.

(c) Construct a histogram based upon this distribution.

(d) From the individual items compute the average daily wage for the 158 employees.

(e) From the frequency distribution calculate the arithmetic mean by the long method (i.e., multiply the midpoint of each class-interval by the corresponding frequency and take the mean of the sum of the products).

(f) Compare the mean calculated in (d) with the mean determined in (e). Why are the two averages not in exact agreement? By what means might the discrepancy in results be reduced?

2. (a) Construct a cumulative table from the distribution secured in problem C1, cumulating the frequencies upward.

(b) Draw an ogive based on the cumulative table.

3. For the distribution secured in problem C1, calculate the

(a) median

(b) the mode

How does the median compare with the median secured from the original data? What assumption is made in regard to the distribution of the items within the class-intervals?

4. From the frequency distribution constructed in problem C1 calculate the

arithmetic mean by the short method. Compare the result with the mean calculated in part (e) of problem C1. Why should the two results be in complete agreement?

5. The following data relate to the operation of Class I steam railways in the United States for the calendar year 1938.

Ratio of Operating Expenses to Operating Revenues for 135 Class I Steam Railroads in the United States for the Year ending December 31, 1938

73.64	81.49	108.66	95.71	83.87	75.62
81.83	106.51	67.59	91.62	80.71	106.55
71.54	68.64	75.30	81.52	71.83	55.40
70.28	75.05	76.43	66.01	92.77	79.52
90.27	84.45	111.85	77.85	77.98	84.89
80.19	75.60	77.93	90.47	67.15	78.68
73.57	85.33	63.59	78.84	71.21	75.82
76.55	71.33	102.21	74.25	73.47	68.05
73.65	60.58	60.09	78.16	49.86	89.09
90.22	80.36	76.00	55.47	83.56	76.84
80.15	86.04	82.55	72.99	93.77	70.35
88.24	99.52	73.44	67.96	80.87	82.04
85.44	74.24	65.96	87.86	77.35	64.92
62.48	71.12	74.09	104.02	95.36	85.96
80.09	87.65	85.82	77.25	81.13	73.51
68.82	75.39	82.85	80.26	82.52	73.19
78.03	78.38	72.08	79.65	71.18	84.35
69.34	57.20	70.13	84.28	118.14	78.74
67.41	70.39	88.86	92.85	88.38	88.63
78.47	64.52	50.70	64.96	84.93	54.56
85.11	80.42	60.73	89.97	80.03	74.90
69.52	111.92	69.58	88.10	73.21	73.35
78.06	99.64	75.16			

Source: U. S. Bureau of Statistics: "52nd Annual Report on the Statistics of Railways in the United States for the Year ended December 31, 1938."

(a) Arrange these items in the form of an array, listing the items in order of size from smallest to largest.

(b) Construct a frequency distribution from the array. Several experimental distributions having different size class-intervals and having different location of class limits should be tested before selecting a final one for presentation.

(c) Construct a frequency polygon based upon this distribution.

(d) From the individual items compute the mean operating ratio for the 135 railroads.

(e) From the frequency distribution calculate the arithmetic mean by the long method (averaging the products of the class midpoints multiplied by the corresponding frequencies). Compare the result with the mean calculated in part (d).

6. (a) Construct a cumulative table from the distribution secured in problem C5, cumulating the frequencies downward.
   (b) Draw an ogive based on the cumulative table.

7. For the distribution secured in problem C5, calculate the median and the mode. How does the median compare with the median secured from the original data? What assumptions are involved regarding the distribution of items within class-intervals in calculating the median and mode from a frequency distribution?

8. From the frequency distribution constructed in problem C5 calculate the arithmetic mean by the short method. Why should this result be in agreement with the mean determined in C5(e)?

9. The following tabulation gives the per cent of depreciation of physical property in 1929 of 188 utility companies.

Per Cent Depreciation of Property of 188 Electric and Gas Operating Utility Companies (subsidiaries of registered public utility holding companies), 1939

1.40	1.64	2.32	1.41	2.55	2.57	2.56	1.84
0.93	1.51	1.86	2.62	2.01	1.02	2.25	0.95
2.03	1.57	2.12	2.04	0.99	1.06	0.46	1.05
0.95	2.57	1.48	2.78	1.31	2.25	2.86	1.74
3.78	2.78	1.88	1.64	1.79	1.94	2.15	1.64
3.08	1.93	2.58	1.83	1.34	2.06	1.75	1.82
1.94	2.49	2.53	2.09	1.82	2.73	1.34	1.43
1.94	2.31	2.03	1.74	2.28	1.25	2.74	2.14
3.13	1.23	1.41	1.08	1.34	1.67	1.93	3.28
2.07	1.05	1.49	1.32	2.41	2.38	0.70	1.08
1.53	1.78	2.12	1.66	1.77	2.71	2.53	2.20
2.58	2.69	2.13	1.35	3.27	2.72	2.38	1.60
2.55	1.23	1.58	2.25	0.95	1.76	1.89	1.27
1.29	1.04	1.01	1.52	2.65	0.82	2.90	2.44
2.50	1.00	0.77	1.24	2.05	1.86	2.38	2.50
1.10	2.24	2.20	1.26	0.65	1.67	1.99	1.74
2.80	2.41	1.54	4.53	1.29	1.41	1.94	2.05
0.83	1.45	2.96	2.33	1.49	2.18	2.24	1.63
1.82	2.53	2.71	1.63	2.04	2.83	1.55	2.42
2.29	1.09	2.25	2.34	1.61	0.76	0.88	1.11
1.65	3.77	2.13	1.63	2.78	1.77		
2.26	2.22	3.18	2.24	3.25	2.05		
1.08	2.76	3.20	1.94	2.61	2.06		
1.90	2.31	2.20	1.59	2.31			
2.39	2.74	2.30	1.80	2.50			

Source: "Financial Statistics for Electric and Gas Subsidiaries of Registered Public-Utility Companies, Year 1939"—Report of Public Utilities Division, Securities and Exchange Commission, Washington, D. C.

(a) Arrange these items in the form of an array, listing the items in order of size from smallest to largest.
(b) Construct a frequency distribution from the array. Several experimental distributions having different size class-intervals and having different location of class-limits should be tested before selecting the final one.
(c) Construct a frequency polygon based upon this distribution.
(d) From the individual items compute the mean per cent of depreciation for the 188 companies.
(e) From the frequency distribution calculate the arithmetic mean by the long method (averaging the products of the class midpoints multiplied by the corresponding frequencies). Compare the result with the mean calculated in part (d).

10. (a) Construct a cumulative table from the distribution secured in problem C9, cumulating the frequencies downward.
(b) Draw an ogive based on the cumulative table.

11. For the distribution secured in problem C9, calculate the median and mode. How does the median compare with the median secured from the original data? Under what circumstances would the two sets of averages be in complete agreement?

12. From the frequency distribution constructed in problem C9 calculate the arithmetic mean by the short method. Why is this result in agreement with the mean determined in C9(e)?

(Note to instructor: Additional data for use in assignments covering the above procedures are given in Tables 3 and 4 of Appendix A.)

13. The following distribution gives the annual net income for 162 white families in Norfolk-Portsmouth, Virginia in 1934–1936.

Annual Net Income	Number of Families
$600– 899	10
900–1199	23
1200–1499	40
1500–1799	32
1800–2099	28
2100–2399	20
2400–2699	4
2700–2999	2
3000–3299	1
3300–3599	2
Total	162

Source: U. S. Bureau of Labor Statistics: *Money Disbursements of Employed Wage Earners and Clerical Workers in Twelve Cities of the South, 1934–1936.* Bulletin 640

For this distribution compute the following measures:
(a) range
(b) arithmetic mean (by the short method)
(c) quartile deviation (semi-interquartile range). The value of K (the value of a point on the scale half-way between the first and third quartiles) should be included with the calculation of the quartile deviation.
(d) mean deviation from the median (or from the mean at the instructor's direction)
(e) standard deviation
(f) Pearson's coefficient of skewness
(g) Bowley's coefficient of skewness

14. Interpret the statistical measures computed in problem C13 with direct reference to the data being described. What limitations to the interpretations are suggested by the techniques employed in the calculation of the various measures?

15. The following distribution gives the annual net income for 419 white families in Baltimore, Maryland in 1934–1936.

Annual Net Income	Number of Families
$300–$599	4
600– 899	45
900–1199	95
1200–1499	120
1500–1799	67
1800–2099	51
2100–2399	17
2400–2699	9
2700–2999	5
3000–3299	3
3300–3599	2
3600–3899	1
Total	419

Source: U. S. Bureau of Labor Statistics: *Money Disbursements of Employed Wage Earners and Clerical Workers in Twelve Cities of the South, 1934–1936.* Bulletin 640

Describe the above distribution with the following measures:
(a) arithmetic mean
(b) median
(c) mode
(d) standard deviation
(e) Pearson's coefficient of skewness
Interpret these measures with direct reference to the distribution.

16. The following distribution gives the number of workers in the explosives industry in the United States according to weekly earnings in October 1937.

Weekly Earnings	Number of Employees
Under $5	12
$5 and under $10	37
$10 and under $15	28
$15 and under $20	155
$20 and under $25	534
$25 and under $30	849
$30 and under $35	1007
$35 and under $40	580
$40 and under $45	366
$45 and under $50	144
$50 and under $55	61
$55 and under $60	21
$60 and under $65	11
$65 and under $70	6
$70 and under $75	3
	3814

Source: Monthly Labor Review, August, 1938, p. 392

Describe the distribution with the following measures:
(a) arithmetic mean, median and mode
(b) range, quartile deviation and standard deviation
(c) Pearson's and Bowley's coefficients of skewness

17. A study of the duration of business cycles, according to approximate duration in years, in five English-speaking countries and twelve non-English speaking countries revealed the following data.

Cycle Duration in Years	Five English-Speaking Countries	Twelve Non-English-Speaking Countries
1	1	2
2	9	8
3	13	17
4	12	13
5	13	10
6	10	12
7	6	11
8	3	9
9	2	5
10	3	3
11	1	2
12	0	1
Totals	73	93

Source: Mitchell, W. C., *Business Cycles,* 1927 ed., p. 400

For the purpose of comparing the characteristics of the two distributions, compute for each series the following measures:
(a) arithmetic mean
(b) standard deviation
(c) coefficient of variation
(d) Pearson's coefficient of skewness
Which series shows the longest average cycle? Which series has the greatest absolute variation? the greatest relative variation?

18. The following distribution gives the number of skilled employees in 68 furniture factories in the United States in 1936 classified according to annual earnings.

Annual Earnings	Number of Employees
$ 0–$199	95
200– 399	157
400– 599	175
600– 799	249
800– 999	523
1000–1199	719
1200–1399	728
1400–1599	446
1600–1799	263
1800–1999	157
2000–2199	77
2200–2399	33
2400 and over	33
Total	3655

Source: Monthly Labor Review, April, 1939, p. 788

For the above distribution compute the following descriptive measures:
(a) median and mode
(b) quartile deviation
(c) Bowley's coefficient of skewness
Interpret the results.

19. A study of the movement of commodity prices from 1891 to 1897 (Mills, F. C., *The Behavior of Prices*) involved the calculation of 195 price relatives (Table 3, Appendix A). A frequency distribution of these relatives in natural numbers and in logarithms is given below. For the purpose of calculating the geometric mean and logarithmic standard deviation, the class intervals have been set up with constant width in terms of logarithms.

Class-interval (natural numbers)	Class-interval (logarithms)	Mid-point (logarithms)	Frequency
35.00 to 39.59	1.54407–1.59757	1.57083	1
39.59 to 44.78	1.59758–1.65108	1.62434	3
44.78 to 50.65	1.65109–1.70459	1.67785	6

50.65 to 57.29	1.70460–1.75810	1.73136	7
57.29 to 64.81	1.75811–1.81161	1.78487	9
64.81 to 73.31	1.81162–1.86512	1.83838	32
73.31 to 82.92	1.86513–1.91863	1.89189	42
82.92 to 93.79	1.91864–1.97214	1.94540	53
93.79 to 106.09	1.97215–2.02565	1.99891	30
106.09 to 120.00	2.02566–2.07916	2.05242	11
120.00 to 135.73	2.07917–2.13267	2.10593	1
Total			195

Calculate the geometric mean and the standard deviation in terms of relatives. Interpret these two measures.

20. Using the data of problem C5, construct a frequency distribution with logarithmic class intervals. Compute and interpret the geometric mean and standard deviation in terms of relatives. Construct a frequency polygon from the distribution using natural number class-intervals on semilogarithmic paper or logarithm class-intervals on arithmetic paper.

21. A New York City typing bureau employing 79 typists conducted tests to determine the average time required for each girl to type a standard page of 300 words. The results of the examination are given in the distribution below with class-intervals in minutes per page and in reciprocal intervals of constant width

Class-interval (minutes per page)	Class-interval (reciprocals)	Mid-point (reciprocals)	Frequency
6.21 to 5.92	.1610–.16899	.1650	4
5.92 to 5.65	.1690–.17699	.1730	8
5.65 to 5.41	.1770–.18499	.1810	12
5.41 to 5.18	.1850–.19299	.1890	16
5.18 to 4.98	.1930–.20099	.1970	14
4.98 to 4.78	.2010–.20899	.2050	11
4.78 to 4.61	.2090–.21699	.2130	7
4.61 to 4.44	.2170–.22499	.2210	3
4.44 to 4.29	.2250–.23299	.2290	3
4.29 to 4.15	.2330–.24099	.2370	1
Total			79

Calculate the harmonic mean and the standard deviation in terms of reciprocals. Interpret these two measures.

D. INDEX NUMBERS OF PRICES; CONSTRUCTION

1. The Bureau of Labor Statistics, U. S. Department of Labor, reports the following quotations on the price of raw silk for the years 1921–1940.

Raw Silk Prices, Wholesale, Japanese, 13–15 denier, (N. Y.)
(monthly averages)

Year	Dollars per pound	Year	Dollars per pound
1921	6.574	1931	2.401
1922	7.648	1932	1.561
1923	8.653	1933	1.612
1924	6.248	1934	1.298
1925	6.574	1935	1.633
1926	6.194	1936	1.777
1927	5.443	1937	1.878
1928	5.072	1938	1.706
1929	4.933	1939	2.178
1930	3.415	1940	2.781

(a) Reduce these prices to relatives on a 1926 base.

(b) Alternative assignment: Reduce these prices to relatives using the average of 1935–1939 as the base.

2. Data on the wholesale price of winter wheat flour, as reported by the United States Bureau of Labor Statistics, are given in Table 9 of Appendix A for the years 1921–1940.

(a) Reduce these prices to relatives on a 1926 base.

(b) Alternative assignment: Reduce these prices to relatives using the average month of 1923–1925 as the base.

3. Data on the prices received by farmers for raw cotton as reported by the Agricultural Marketing Service, U. S. Department of Agriculture, are listed in Table 9 of Appendix A for the years 1921–1940.

(a) Reduce these prices to relatives on a 1928 base.

(b) Alternative assignment: Reduce these prices to relatives using the average of 1928–1930 as the base.

4. The *Survey of Current Business*, U. S. Department of Commerce, publishes the following data on the prices and consumption of the four leading textiles.

Average Monthly Textile Prices, 1930–1940, in Dollars per Pound

Textile*	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939	1940
Cotton	.135	.085	.064	.087	.123	.119	.121	.114	.087	.095	.104
Rayon	1.06	.76	.66	.61	.59	.57	.59	.62	.52	.52	.53
Wool	.76	.62	.46	.68	.82	.75	.92	.99	.70	.83	.95
Silk	3.415	2.401	1.561	1.612	1.298	1.633	1.777	1.878	1.706	2.718	2.781

* The above quotations are for the following grades:
Cotton: wholesale, middling, upland (New York)
Rayon: wholesale, 150 denier, first quality (New York)
Wool: raw, wholesale, territory, fine, staple, scoured
Silk: wholesale, raw, Japanese, 13–15 (New York)

Average Monthly Textile Consumption in the United States, 1930–1940, in Millions of Pounds

Textile	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939	1940
Cotton	2,689	2,722	2,509	3,105	2,710	2,825	3,552	3,709	2,951	3,686	4,046
Rayon	118	158	153	213	196	253	298	267	274	363	389
Wool	263	311	230	318	230	417	406	381	285	396	409
Silk	77	79	73	62	61	66	60	56	55	51	50

(a) Using 1936 as the base year, construct textile price indexes for the years 1930–1936 of the following types:
   (1) Simple aggregate as a relative
   (2) Simple mean of relatives
   (3) Simple median of relatives
   (4) Simple geometric mean of relatives
   (5) Weighted aggregate as a relative (using base year weights)
   (6) Weighted mean of relatives (using given year weights)
   (7) Weighted median of relatives (using given year weights)
   (8) Weighted geometric mean of relatives (using given year weights)

(b) Alternative assignment: Using the average of 1933–1935 as the base, construct textile price indexes for the years 1932–1940 of each type listed in (a).

(c) Alternative assignment: Using the average of 1936–1938 as the base, construct textile price indexes for the years 1930–1940 of each type listed in (a). For parts 6–8 use base year weights, and for part 5 use given year weights.

5. Data on the average monthly price per bushel and on the production of six selected grains for the years 1925–1940 are listed in Tables 10 and 11 of Appendix A. Using the average of 1933–1935 as the base, construct grain price indexes of the types listed in D4 (a) for the years 1930–1940.

6. Using the data indicated in D5, with 1925 as the base year, construct for the years 1925–1933 grain price indexes of the types listed in D4 (a).

7. Using the data indicated in D5, with 1935–1938 = 100, construct grain price indexes of the types listed in D4 (a).

8. Data covering average retail prices of five selected articles of food for the years 1923–1937, together with apparent per capita consumption estimates, are given in Table 12 of Appendix A.

(a) Using 1923 as the base year, construct retail food price indexes for the years 1923–1933 of the following types:
  (1) Simple aggregate as a relative
  (2) Simple mean of relatives
  (3) Simple median of relatives
  (4) Simple geometric mean of relatives
  (5) Weighted aggregate as a relative (using the estimated weights)
  (6) Weighted mean of relatives (using 1926–1928 values as weights)
  (7) Weighted median of relatives (using 1926–1928 values as weights)
  (8) Weighted geometric mean of relatives (using 1926–1928 values as weights)

9. Using the data mentioned in D8, construct retail food price indexes for the years 1929–1937, of the types listed in the same problem, using 1930–1932 as the base.

10. Using the data mentioned in D8, construct retail food price indexes for the years 1926–1935 of the types listed in the same problem, setting 1936 equal to 100.

11. Using the data of problem D4, compute for the years 1930–1933 "ideal" indexes using 1933 as the base year. What especial merit does this type of index possess? What effect does the change of weights from year to year have on the comparability of the indexes?

12. Using the data of problem D5, compute for the years 1930–1935 "ideal" indexes using 1930 as the base year. Discuss the merits and limitations of this type of index.

13. Data on the wholesale price and consumption of eggs, potatoes, beef and veal, and butter are given in Table 13 of Appendix A. Construct wholesale food price indexes of the types and for the years assigned by the instructor.

14. Using the data indicated in problem D11, compute for the same years price indexes based on the substitute formula for the "ideal" index (weighted aggregates as relatives using combined quantities as weights).

E. INDEX NUMBERS OF PRICES; APPLICATIONS

1. Retail food price indexes are computed by both the National Industrial Conference Board and the United States Bureau of Labor Statistics. Restate one of index series on the base used by the other organization.

Average Monthly Retail Food Price Indexes, 1914–1940

Year	National Industrial Conference Board (mo. av. 1923 = 100)	U. S. Bureau of Labor Statistics (mo. av. 1935–1939 = 100)
1914	66.1	81.8
1915	64.8	80.9
1916	71.5	90.8
1917	94.3	116.9
1918	111.7	134.4
1919	120.9	152.1
1920	136.2	168.5
1921	103.5	128.6
1922	96.8	120.3
1923	100.0	124.0
1924	99.1	122.8
1925	107.2	132.9
1926	110.6	137.4
1927	107.0	132.3
1928	105.6	130.8
1929	106.9	132.5
1930	101.7	126.0
1931	83.7	103.9
1932	69.7	86.5
1933	67.8	84.1
1934	75.3	93.7
1935	82.1	100.4
1936	83.9	101.3
1937	86.9	105.3
1938	80.7	97.8
1939	78.6	95.2
1940	77.7	96.7

Source: Survey of Current Business

2. Using the United States Department of Labor indexes of wholesale commodity prices given in Table 1 of Appendix A, restate the series on a 1935–1939 base.

3. Construct link indexes from the following price data:

Average Monthly Wholesale Prices of Linseed Oil in New York, 1920–1940, in Dollars per Pound

1920	.195	1923	.132
1921	.093	1924	.131
1922	.113	1925	.139

1926	.112	1934	.094
1927	.105	1935	.094
1928	.100	1936	.098
1929	.123	1937	.108
1930	.125	1938	.090
1931	.084	1939	.092
1932	.063	1940	.096
1933	.091		

Source: Survey of Current Business

4. Using the data on wheat flour prices (Table 9 of Appendix A), construct link indexes.

5. Using the data on prices received by farmers for raw cotton (Table 9 of Appendix A), construct link indexes.

6. Using the link indexes calculated in problem E3, E4 or E5 above, construct chain indexes on a 1926 base and compare with fixed base indexes calculated from the original data.

7. Using the building materials price index series given below, construct a purchasing power index series in the wholesale market for building materials on the base 1926.

United States Department of Labor Building Materials Price Indexes at Wholesale (monthly averages), 1920–1940 (1926 = 100)

1920	150.1	1931	79.2
1921	97.4	1932	71.4
1922	97.3	1933	77.0
1923	108.7	1934	86.2
1924	102.3	1935	85.3
1925	101.7	1936	86.7
1926	100.0	1937	95.2
1927	94.7	1938	90.3
1928	94.1	1939	90.5
1929	95.4	1940	94.6
1930	89.9		

Source: Survey of Current Business

8. Using the retail price index series given below, construct purchasing power indexes of the "retail dollar" on the base 1935.

Fairchild's Retail Price Indexes, 1931–1940 (monthly averages) Dec. 31, 1930 = 100

1931	90.9	1936	88.9
1932	75.8	1937	95.1
1933	77.5	1938	89.7
1934	88.3	1939	89.9
1935	86.6	1940	93.0

Source: Survey of Current Business

9. Using the U. S. Bureau of Labor Statistics indexes of wholesale commodity prices (1926 = 100) given in Table 1 of Appendix A, construct purchasing power indexes of the "wholesale dollar" on the base 1926 for the years 1925–1940. Give an exact interpretation of the computed indexes.

10. Determine the "real price"* of crude rubber in terms of 1926 dollars by deflating the following data, using the U. S. Bureau of Labor Statistics wholesale commodity price index series (Table 1 of Appendix A). Give a precise interpretation of the deflated series.

Wholesale Price of Crude Rubber (smoked sheets, New York) (dollars per pound)

1930	.119	1936	.164
1931	.061	1937	.194
1932	.034	1938	.147
1933	.060	1939	.176
1934	.129	1940	.202
1935	.124		

Source: Survey of Current Business

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* Statisticians contrast money wages or prices with "real wages" or "real prices," meaning by the latter, wages or prices corrected for changes in the purchasing power of money in stated markets. The reference is always to a given base period.

11. Determine the "real price" of linseed oil for the years 1920–1940 (data of problem E3) using the U. S. Bureau of Labor Statistics wholesale price index (Table 1, Appendix A) for the adjustment. Give a precise interpretation of the deflated series.

12. Determine the "real price" of wheat flour for the years 1921–1940 (data from Table 9, Appendix A) using the U. S. Bureau of Labor Statistics wholesale commodity price series for the adjustment. Interpret the deflated series fully.

13. Deflate the series below for the effects of changes in the cost of construction. How do you interpret the adjusted data?

	Contracts Awarded for Total Construction in 37 States, (millions of dollars)	Construction Cost Index, all types (monthly average 1913 = 100)
	Monthly averages for the year	
1925	500.5	206.4
1926	531.7	208.4
1927	525.3	205.6
1928	552.4	207.2
1929	479.2	207.0
1930	376.9	201.6
1931	257.7	178.7
1932	112.6	156.6
1933	104.6	172.9
1934	128.6	198.7

1935	153.7	196.7
1936	222.9	208.1
1937	242.8	236.3
1938	266.4	235.4
1939	295.9	235.8
1940	333.7	243.0

Source: Contracts awarded compiled by the F. W. Dodge Corporation; Construction cost index compiled by the *Engineering-News Record*

14. Deflate the data showing indicated expenditures for building construction (Table 14, Appendix A) for the years 1929–1940 using the construction cost index series given in problem E13. Interpret precisely the adjusted construction series. The interpretation will be made clearer if the construction cost index series is shifted to the same base used for the construction permits data.

15. Deflate the index series on department store sales (Table 1, Appendix A) using Fairchild's retail price index (problem E8 above) for the adjustment. The retail price index should be shifted to a 1923–1925 base for a clearer understanding of the meaning of the deflated sales data.

16. The following data are factory average hourly earnings as compiled by the National Industrial Conference Board based on 25 industries in the United States for the years 1921–1940. Using the cost of living index series compiled by the same organization, determine the "real wage rate" for factory workers in terms of 1923 living costs.

Year	Average Hourly Earnings of Factory Workers in 25 Industries (monthly averages)	Cost of Living Index 1923 = 100 (monthly averages)
1921	$.524	102.3
1922	.494	97.4
1923	.541	100.0
1924	.562	101.3
1925	.561	103.7
1926	.568	104.3
1927	.576	102.0
1928	.579	100.6
1929	.590	100.1
1930	.589	96.7
1931	.564	87.2
1932	.498	77.9
1933	.491	74.9
1934	.580	79.4
1935	.599	82.6
1936	.619	84.8
1937	.695	88.5
1938	.716	86.4
1939	.720	85.2
1940	.739	85.3

Source: Survey of Current Business

17. Determine the "real weekly earnings" for factory employees in New York State, applying the cost of living index series given in problem E16. Give a precise interpretation of the adjusted series.

Factory Average Weekly Earnings of New York State Employees 1920–1940
(monthly average 1925–27 = 100)

1920	97.5	1931	91.2
1921	89.1	1932	78.5
1922	86.7	1933	75.4
1923	94.4	1934	80.1
1924	95.9	1935	84.1
1925	97.9	1936	87.5
1926	100.6	1937	94.5
1927	101.5	1938	91.7
1928	102.0	1939	94.7
1929	103.9	1940	98.4
1930	99.5		

Source: New York State Department of Labor

18. Determine the "real hourly wages" for employees of Class I steam railways, applying the cost of living index series given in E16. Interpret the calculated results.

Average Hourly Wages of Employees of Class I Steam Railways,
1921–1940 (dollars per hour)

1921	.600	1931	.669
1922	.593	1932	.616
1923	.592	1933	.609
1924	.602	1934	.617
1925	.610	1935	.668
1926	.618	1936	.674
1927	.629	1937	.693
1928	.639	1938	.730
1929	.651	1939	.731
1930	.661	1940	.734

Source: Interstate Commerce Commission Reports

19. *United States Bureau of Labor Statistics Wholesale Commodity Price Indexes According to Economic Classes, 1913–1940 (monthly averages)*
1926 = 100

Year	Combined index	Economic classes		
		Finished products	Raw materials	Semi-manufactures
1913	69.8	69.4	68.8	74.9
1914	68.1	67.8	67.6	70.0

1915	69.5	68.9	67.2	81.2
1916	85.5	82.3	82.6	118.3
1917	117.5	109.2	122.6	150.4
1918	131.3	124.7	135.8	153.8
1919	138.6	130.6	145.9	157.9
1920	154.4	149.8	151.8	198.2
1921	97.6	103.3	88.3	96.1
1922	96.7	96.5	96.0	98.9
1923	100.6	99.2	98.5	118.6
1924	98.0	96.3	97.6	108.7
1925	103.5	100.6	106.7	105.3
1926	100.0	100.0	100.0	100.0
1927	95.4	95.0	96.5	94.3
1928	96.7	95.9	99.1	94.5
1929	95.3	94.5	97.5	93.9
1930	86.4	88.0	84.3	81.8
1931	73.0	77.0	65.6	69.0
1932	64.8	70.3	55.1	59.3
1933	65.9	70.5	56.5	65.4
1934	74.9	78.2	68.6	72.8
1935	80.0	82.2	77.1	73.6
1936	80.8	82.0	79.9	75.9
1937	86.3	87.2	84.8	85.3
1938	78.6	82.2	72.0	75.4
1939	77.1	80.4	80.2	77.0
1940	78.5	81.5	71.9	79.1

Source: U. S. Bureau of Labor Statistics

Compare the movement of prices in the economic classes as given above

(a) graphically, by plotting the three series to the same scale on coordinate paper and,

(b) by studying the three series in regard to timing and amplitude of movement.

Which of the three series dominates the combined index?

20. The above index series (problem E19) are of the weighted aggregate type, the weights being determined according to the quantities marketed in selected years. Justify the choice of weights.

Under what circumstances would production or consumption quantities be more uitable?

21. Discuss the problem of price index construction from the point of view of

(a) the number of individual price series included in the composite
(b) the base year employed
(c) the choice of weights
(d) the formula used in combining the individual series
(e) the length of period covered by the composite
(f) the maintenance of uniform quality at different dates

F. ANALYSIS OF TIME SERIES; SECULAR TREND

1. The following data indicate the estimated cotton consumption in the United States for the years 1900–1940.

Year	Cotton Consumption (in millions of bales)	Year	Cotton Consumption (in millions of bales)
1900	3.69	1921	5.41
1901	3.60	1922	6.09
1902	4.08	1923	6.52
1903	4.19	1924	5.52
1904	3.98	1925	6.43
1905	4.52	1926	6.68
1906	4.91	1927	7.41
1907	4.99	1928	6.57
1908	4.54	1929	7.05
1909	5.09	1930	5.38
1910	4.62	1931	5.44
1911	4.50	1932	5.02
1912	5.13	1933	6.21
1913	5.58	1934	5.42
1914	5.45	1935	5.65
1915	6.01	1936	7.10
1916	6.62	1937	7.42
1917	6.82	1938	5.90
1918	6.18	1939	7.37
1919	5.92	1940	8.06
1920	5.84		

Source: U. S. Department of Commerce, Bureau of the Census

(a) Plot the above data on arithmetic graph paper.
(b) Determine by inspection the part of the forty-one year period which can be designated with a straight line secular trend. Freehand, draw in a linear trend.
(c) Determine from the graph the "a" (y-intercept) and "b" (slope) values for the linear trend equation $y=a+bx$.
(d) What criteria were used in
  (1) selecting the period for which the trend was estimated?
  (2) placing the linear trend line as drawn freehand?
(e) What merits and limitations has the freehand method as a means of approximating trend?

2. Using the data on steel production in the United States in Table 2 of Appendix A, perform the operations outlined in F1.

3. Using the data on cement production in the United States in Table 2 of Appendix A, for the years 1910–1940, perform the operations outlined in F1.

4. Using the data on cotton consumption (given in F1 above) for the years

1911–1940, approximate a linear trend by the method of semi-averages. Plot the trend approximation on an arithmetic grid showing the original data.

5. Using the data of problem F1 above, approximate a linear trend by the method of semi-medians. Plot the trend approximation on an arithmetic grid showing the original data. Under what circumstances does this method of trend estimation give a better approximation than the semi-averages method?

6. Repeat the instructions in problem F4 using the data for steel production (Table 2 of Appendix A) for the years 1911–1940.

7. Repeat the instructions in problem F5 using the data for steel production (Table 2 of Appendix A) for the years 1911–1940.

8. Repeat the instructions in problem F4 using the data for cement production (Table 2 of Appendix A) for the years 1904–1940.

9. Repeat the instructions in problem F5 using the data for cement production (Table 2 of Appendix A) for the years 1904–1940.

10. Plot the data on cotton consumption given in problem F1 on an arithmetic chart.

(a) By inspection determine the average length of cycle in the series.
(b) Calculate and plot a centered moving-average approximation of trend using the average length of cycle for the length of average employed.
(c) Calculate and plot a centered moving-average approximation of trend using an average twice the length of average cycle.
(d) What advantages and disadvantages are suggested by the moving-average trend approximation method?

11. Repeat the instructions of problem F10 using the data for bituminous coal production (Table 2 of Appendix A) for the years 1900–1940.

12. Repeat the instructions of problem F10 using the data for crude petroleum production for the years 1900–1940.

13. By the least squares method, calculate and plot the linear equation $y = a + bx$ from the following 9 points.

y	x
2	1
4	2
3	3
5	4
4	5
6	6
5	7
7	8
6	9

(a) Interpret the values of "a" and "b" from the equation.
(b) From the equation determine the corresponding y values for x values of 10 and 20.

14. By the least squares method, calculate and plot the linear equation $y = a + bx$ from the following 9 points.

y	x
6	−4
10	−3
7	−2
11	−1
8	0
12	+1
9	+2
13	+3
10	+4

(a) Interpret the values of "a" and "b" from the equation. Determine the corresponding y values for x values of −6 and +6.

15. The following table gives the average monthly imports of wood pulp into the United States for the years 1920–1940.

Year	Average Monthly Imports (million sh. tons)	Year	Average Monthly Imports (million sh. tons)
1920	75.5	1931	133.0
1921	58.1	1932	123.5
1922	104.9	1933	161.8
1923	115.3	1934	150.5
1924	126.9	1935	161.1
1925	138.7	1936	189.8
1926	144.5	1937	199.6
1927	139.9	1938	142.5
1928	146.8	1939	168.9
1929	157.2	1940	102.0
1930	152.5		

Source: Survey of Current Business, Oct. 1940; March, 1941

(a) Plot the data on arithmetic graph paper.
(b) Calculate and plot the linear least squares equation of trend using 1920 as the origin year.
(c) Interpret specifically the values of "a" and "b" from the equation.
(d) Extrapolate the equation for five years following.
(e) What assumptions are implied in such a trend projection?

16. Repeat the instructions of problem F15 using the year 1930 as the origin year.

17. The following table gives the total annual domestic demand for motor fuel in the United States for the years 1920–1940.

Year	Total Motor Fuel Demand (million bbls.)	Year	Total Motor Fuel Demand (million bbls.)
1920	102.2	1931	403.4
1921	108.5	1932	377.8
1922	129.1	1933	380.5
1923	158.2	1934	410.3
1924	186.8	1935	434.8
1925	226.0	1936	481.6
1926	199.6	1937	519.3
1927	331.6	1938	523.0
1928	376.0	1939	555.5
1929	376.0	1940	589.4
1930	394.8		

Source: Survey of Current Business

(a) Using the above data, repeat the instructions of problem F15 with 1920 as the origin year.

18. Using the data of problem F17, repeat the instructions of problem F15 using 1930 as the origin year.

19. The following table gives the production of crude petroleum in the United States for the years 1920–1940.

Year	Average Monthly Production (in million barrels)	Year	Average Monthly Production (in million barrels)
1920	36.9	1931	70.9
1921	39.3	1932	65.4
1922	46.5	1933	75.5
1923	61.0	1934	75.7
1924	59.5	1935	83.1
1925	63.6	1936	91.6
1926	64.2	1937	106.6
1927	75.1	1938	101.2
1928	75.1	1939	105.4
1929	83.9	1940	112.7
1930	74.8		

Source: Survey of Current Business

(a) Plot the data on arithmetic graph paper.
(b) Calculate and plot the linear least squares equation of trend using 1920 as the origin year.
(c) Interpret specifically the values of "*a*" and "*b*" from the equation.
(d) Extrapolate the equation for five years following.
(e) What assumptions are implied in such a trend projection?

20. Repeat the instructions of problem F19 using the year 1930 as the origin.

21. The following table gives the production of wood pulp (sulphate) in the United States for the years 1920–1940.

Year	Average Monthly Production (in thousand short tons)	Year	Average Monthly Production (in thousand short tons)
1920	15.7	1931	86.1
1921	11.5	1932	85.7
1922	20.3	1933	104.9
1923	26.0	1934	103.9
1924	25.2	1935	122.3
1925	34.1	1936	149.6
1926	43.3	1937	178.3
1927	50.3	1938	203.6
1928	64.5	1939	249.3
1929	76.5	1940	310.2
1930	79.1		

Source: Survey of Current Business

(a) Plot the data on arithmetic graph paper.
(b) Calculate and plot the equation of trend of the "parabolic" type $y=a+bx+cx^2$, using 1920 as the origin year.
(c) Extrapolate the equation for the five following years.

22. Repeat the instructions of problem F21 using 1930 as the origin year.

23. The following table gives the value of domestic money orders paid out in fifty cities of the United States for the years 1915–1940.

Year	Average Monthly Value of Money Orders Paid Out (in millions of dollars)	Year	Average Monthly Value of Money Orders Paid Out (in millions of dollars)
1915	34.8	1928	86.3
1916	40.6	1929	91.0
1917	44.9	1930	84.6
1918	50.6	1931	79.2
1919	65.4	1932	75.1
1920	72.4	1933	91.4
1921	64.8	1934	91.8
1922	68.5	1935	93.9
1923	78.9	1936	102.1
1924	84.5	1937	107.6
1925	81.3	1938	100.9
1926	87.3	1939	100.6
1927	88.2	1940	105.0

Source: Survey of Current Business

(a) Plot the data on arithmetic graph paper.

(b) Calculate and plot the equation of trend of the "parabolic" type $y=a+bx+cx^2$ using 1915 as the origin year.

(c) Extrapolate the equation for the five following years.

24. Repeat the instructions of problem F23 using January 1, 1928, as origin.

25. The following table gives the volume of gold receipts (unrefined) at United States mints for the years 1920–1940.

Year	Volume of Gold Received at U. S. Mints (monthly aves.) (thousand fine ounces)	Year	Volume of Gold Received at U. S. Mints (monthly aves.) (thousand fine ounces)
1920	62.4	1931	109.5
1921	65.3	1932	121.0
1922	70.4	1933	127.5
1923	67.5	1934	107.9
1924	70.5	1935	137.6
1925	68.8	1936	193.7
1926	67.2	1937	221.7
1927	80.9	1938	245.2
1928	81.6	1939	262.7
1929	81.6	1940	300.2
1930	99.4		

Source: Survey of Current Business

(a) Plot the data on arithmetic graph paper.

(b) Calculate and plot the equation of trend of the "parabolic" type $y=a+bx+cx^2$ using 1920 as the origin year.

(c) Extrapolate the equation for the following five years. What obvious fallacies are implied in such an extrapolation of this series?

26. Repeat the instructions of problem F25 using 1930 as the origin year.

27. The following table gives the production of women's shoes in the United States for the years 1921–1940.

Year	Monthly Average Production (in million pairs)	Year	Monthly Average Production (in million pairs)
1921	8.46	1931	9.38
1922	8.78	1932	9.50
1923	9.14	1933	10.90
1924	8.68	1934	11.09
1925	8.73	1935	12.10
1926	9.20	1936	13.49
1927	9.69	1937	12.47
1928	10.31	1938	12.31
1929	10.94	1939	13.98
1930	9.39	1940	12.49

Source: Survey of Current Business

(a) Plot the above data on semilogarithmic paper.

(b) Calculate and plot the linear least squares equation of trend of the type $\log y = \log a + x(\log b)$, using January 1, 1931 as the origin date.

(c) Interpret specifically the values of "a" and "b" in the equation when stated in its arithmetic equivalent ($y = ab^x$).

(d) Extrapolate the trend equation for the five following years.

28. Repeat the instructions of problem F27 using 1921 as the origin date.

29. The following table gives the exports of newsprint from Canada for the years 1914–1940.

Year	Monthly Average Exports (in thousand short tons)	Year	Monthly Average Exports (in thousand short tons)
1914	29.8	1928	183.9
1915	35.0	1929	209.6
1916	43.8	1930	194.4
1917	49.7	1931	167.4
1918	53.0	1932	148.1
1919	59.0	1933	153.2
1920	63.5	1934	201.2
1921	59.1	1935	214.6
1922	80.0	1936	249.4
1923	94.8	1937	287.9
1924	101.6	1938	202.1
1925	116.8	1939	221.6
1926	144.3	1940	270.3
1927	156.8		

Source: Survey of Current Business

(a) Plot the data on semilogarithmic paper.

(b) Calculate and plot the equation of trend of the type $\log y = \log a + x(\log b)$, using 1927 as the origin year.

(c) Interpret specifically the value of "a" and "b" in the equation when stated in its arithmetic equivalent ($y = ab^x$).

(d) Extrapolate the equation for the five following years.

30. Repeat the instructions of problem F29 using 1914 as the origin date.

31. The following table gives the refinery stocks of asphalt in the United States for the years 1921–1940.

Year	Average Monthly Stocks (end of month, thousand short tons)	Year	Average Monthly Stocks (end of month, thousand short tons)
1921	113	1924	123
1922	134	1925	145
1923	129	1926	195

1927	212	1934	340
1928	240	1935	401
1929	239	1936	429
1930	297	1937	500
1931	326	1938	580
1932	331	1939	573
1933	276	1940	621

Source: Survey of Current Business

(a) Plot the above data on semilogarithmic paper.
(b) Calculate and plot the linear least squares equation of trend of the type $\log y = \log a + x(\log b)$, using January 1, 1931 as the origin.
(c) Interpret specifically the values of "a" and "b" in the equation when restated in its arithmetic equivalent ($y = ab^x$).
(d) Extrapolate the trend for the five following years.

32. Repeat the instructions of problem F31 using 1921 as the origin date.

33. The following table gives the apparent consumption of creamery butter in the United States for the years 1921–1940.

Year	Apparent Consumption of Butter (monthly averages in millions of pounds)	Year	Apparent Consumption of Butter (monthly averages in millions of pounds)
1921	89.6	1931	142.0
1922	97.6	1932	141.5
1923	105.6	1933	139.4
1924	111.2	1934	146.6
1925	114.7	1935	138.4
1926	122.7	1936	134.8
1927	124.1	1937	137.7
1928	124.2	1938	141.7
1929	129.8	1939	152.5
1930	134.4	1940	151.8

Source: Survey of Current Business

(a) Plot the above data on semilogarithmic paper.
(b) Calculate and plot the curvilinear least squares equation of trend of the type $\log y = \log a + x(\log b) + x^2(\log c)$, using 1921 as the origin year.
(c) Extrapolate the trend equation for the five following years.

34. Repeat the instructions of the above problem using January 1, 1931, as the origin date.

35. The following table gives the apparent consumption of small cigarettes in the United States (based on tax-paid withdrawals) for the years 1915–1940.

Year	Small Cigarette Consumption (in billions) (monthly averages)	Year	Small Cigarette Consumption (in billions) (monthly averages)
1915	1.55	1928	8.83
1916	2.19	1929	9.92
1917	2.90	1930	9.97
1918	3.16	1931	9.45
1919	3.73	1932	8.63
1920	3.72	1933	9.31
1921	4.24	1934	10.47
1922	4.46	1935	11.22
1923	5.37	1936	12.76
1924	5.92	1937	13.55
1925	6.66	1938	13.65
1926	7.45	1939	14.37
1927	8.10	1940	15.06

Source: Survey of Current Business

(a) Plot the above data on semilogarithmic paper.

(b) Calculate and plot the curvilinear least squares trend equation of the type $\log y = \log a + x(\log b) + x^2(\log c)$, using 1915 as the origin year.

(c) Extrapolate the trend equation for the five following years.

36. Repeat the instructions of problem F35 using January 1, 1928, as the date of origin.

37. The following table gives the production of electric power in the United States for the years 1920–1940.

Year	Monthly Average Production (in billion kilowatt hours)	Year	Monthly Average Production (in billion kilowatt hours)
1920	3.61	1930	7.89
1921	3.41	1931	7.56
1922	3.96	1932	6.87
1923	4.63	1933	7.06
1924	4.91	1934	7.57
1925	5.48	1935	8.21
1926	6.14	1936	9.35
1927	6.61	1937	10.15
1928	7.21	1938	9.72
1929	7.99	1939	10.86
		1940	12.08

Source: Survey of Current Business

(a) Plot the above data on semilogarithmic paper.
(b) Calculate and plot the curvilinear least squares equation of trend of the type $\log y = \log a + x(\log b) + x^2(\log c)$, using 1930 as the origin date.
(c) Extrapolate the trend equation for the five following years.

38. Repeat the instructions of problem F37 using 1920 as the origin date.

39. The following table gives the apparent consumption of cheese (based on disappearance into trade) in the United States for the years 1926–1940.

Year	Average Monthly Cheese Consumption (million pounds)	Year	Average Monthly Cheese Consumption (million pounds)
1926	42.5	1934	51.3
1927	40.9	1935	55.9
1928	41.1	1936	57.5
1929	38.3	1937	59.6
1930	40.8	1938	63.4
1931	46.5	1939	64.4
1932	45.7	1940	64.1
1933	47.3		

Source: Survey of Current Business

(a) Plot the data on semilogarithmic paper.
(b) Calculate and plot the equation of trend of the modified exponential type $y = ab^x - K$. (K is approximated empirically by breaking the observed series into three parts representing equal periods of time, determining the means of the observations for each of the three periods, designating the means in chronological order as M_1, M_2 and M_3. The value of K is estimated from the expression

$$\frac{M_2^2 - (M_1 M_3)}{(M_1 + M_3) - 2M_2}.$$

40. The following table gives the imports of coffee into the United States for the years 1929–1940.

Year	Average Monthly Imports of Coffee (thousand bags)	Year	Average Monthly Imports of Coffee (thousand bags)
1929	936	1935	1108
1930	1010	1936	1098
1931	1099	1937	1071
1932	948	1938	1255
1933	1001	1939	1272
1934	962	1940	1292

Source: Survey of Current Business

(a) Repeat the instructions of problem F39.

41. The following table gives the factory production of cheese in the United States for the years 1926–1940.

Year	Average Monthly Cheese Production (factory, American, whole milk) (million pounds)	Year	Average Monthly Cheese Production (factory, American, whole milk) (million pounds)
1926	28.0	1934	36.3
1927	25.6	1935	39.1
1928	27.9	1936	40.6
1929	30.9	1937	41.0
1930	31.6	1938	46.7
1931	31.2	1939	46.3
1932	30.9	1940	49.1
1933	34.1		

Source: Survey of Current Business

(a) Repeat the instructions of problem F39.

42. The following table gives the automobile truck production in the United States for the years 1917–1940.

Year	Annual Total Production (in millions of cars)	Year	Annual Total Production (in millions of cars)
1917	128.2	1929	771.0
1918	227.3	1930	571.2
1919	275.9	1931	416.7
1920	321.8	1932	235.2
1921	148.1	1933	346.5
1922	270.0	1934	575.2
1923	409.3	1935	694.7
1924	416.7	1936	784.6
1925	530.7	1937	893.1
1926	516.9	1938	488.1
1927	464.8	1939	710.5
1928	543.3	1940	777.0

Source: Survey of Current Business

(a) Plot the data on semilogarithmic paper.

(b) Calculate and plot the equation of trend for the Gompertz curve

$$y = ab^{c^x}.$$

(c) Extrapolate the trend equation for the five following years.

43. The data in problem F37 are for electric power production in the United States for the years 1920–1940.

(a) Plot the data on semilogarithmic paper.

(b) Calculate and plot the equation of trend for the Gompertz curve

$$y = ab^{c^x}.$$

(c) Extrapolate the trend equation for the five following years.

44. The data in problem F35 are for small cigarette consumption in the United States.

(a) Plot the data for the years 1917–1940 on semilogarithmic paper.

(b) Calculate and plot the equation of trend for the Gompertz curve

$$y = ab^{c^x}.$$

(c) Extrapolate the trend equation for the five following years.

45. Plot the data of problem F42, F43 or F44 on semilogarithmic paper.

(a) Calculate and plot the logistic trend equation of the type

$$\frac{1}{y} = a + bc^x.$$

(b) Extrapolate the trend equation for the five following years.

G. ANALYSIS OF TIME SERIES; INDEXES OF SEASONAL VARIATION

1. The following table gives the monthly production of boots, shoes and slippers in the United States for the years 1929–1940.

Boot and Shoe Production in the United States, 1929–1940
(Boots, Shoes and Slippers)

Monthly Totals in Millions of Pairs

	1929	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939	1940	Av.
Jan.	27.25	26.53	19.89	21.23	22.72	26.04	29.56	33.36	37.15	25.71	34.16	34.45	28.17
Feb.	27.71	25.90	23.97	25.96	26.38	30.53	30.88	33.05	39.58	30.33	35.93	36.44	30.56
Mar.	30.90	28.63	29.36	30.68	28.58	35.55	34.23	34.83	46.12	37.54	42.92	35.29	34.55
Apr.	29.38	29.00	29.89	25.95	27.63	34.43	34.56	33.40	40.30	33.47	33.06	31.82	31.91
May	29.16	24.51	28.45	22.50	32.97	34.06	31.26	30.26	35.41	30.47	32.71	30.14	30.16
June	28.12	23.90	27.84	23.56	34.86	28.54	27.23	29.37	34.45	26.90	32.31	28.12	28.77
July	30.22	24.12	28.61	20.44	33.75	28.39	32.27	35.68	34.84	30.74	34.21	34.01	30.61
Aug.	36.45	28.43	33.47	30.79	37.02	35.62	37.24	40.67	38.66	42.25	43.95	39.60	37.01
Sept.	34.83	29.33	31.29	33.89	31.23	28.18	33.91	40.98	34.03	38.28	36.81	35.09	33.99
Oct.	37.19	27.73	25.38	33.07	31.46	28.71	35.95	39.92	29.09	35.01	37.27	37.03	33.15
Nov.	27.72	18.54	18.52	25.15	23.70	23.85	27.72	30.34	21.29	30.05	32.13	30.53	25.79
Dec.	22.47	17.54	19.56	20.10	20.10	23.20	28.95	33.38	21.05	29.99	28.69	31.62	24.72
Av.	30.12	25.35	26.35	26.11	29.20	29.76	31.98	34.60	34.33	32.56	35.35	33.68	

Source: Survey of Current Business

(a) Calculate indexes of seasonal variation by the method of averaging the percentages of the original values to the average month of the year. The percentage calculations for the first six years of the period are given in the following table.

(b) Plot the seasonal indexes on an arithmetic grid.

Percentage Relation of Actual Boot and Shoe Production
to Monthly Average of the Year

	1929	1930	1931	1932	1933	1934
Jan.	90.47	104.65	75.48	81.31	77.81	87.50
Feb.	92.00	102.17	90.97	99.43	90.34	102.59
Mar.	102.59	112.94	111.42	117.50	97.88	119.46
Apr.	97.54	114.40	113.43	99.39	94.62	115.69
May	96.81	96.69	107.97	86.17	112.91	114.45
June	93.36	94.28	105.65	90.23	119.38	95.90
July	100.33	95.15	108.58	78.28	115.58	95.40
Aug.	121.02	112.15	127.02	117.92	126.78	119.69
Sept.	115.64	115.70	118.75	129.80	106.95	94.69
Oct.	123.47	109.39	96.32	126.66	107.74	96.47
Nov.	92.03	73.14	70.28	96.32	81.16	80.14
Dec.	74.60	69.19	74.23	76.98	68.84	77.96

2. Using the data on boot and shoe production given in problem G1, compute indexes of seasonal variation by the method of simple averages. For the purpose of adjusting the original averages of the twelve months, the linear least squares equation of trend, calculated from the original monthly data is $y = 28.55 + .037\,x$ (origin at January, 1929).

(a) Plot the seasonal indexes on an arithmetic grid.

3. Using the data on boot and shoe production given in problem G1, compute indexes of seasonal variation by the link-relative method. The link-relatives for the first six years of the period are given in the following table.

(a) Plot the seasonal indexes on an arithmetic grid.

Percentage Relation of Actual Boot and Shoe Production to Preceding Month

	1929	1930	1931	1932	1933	1934
Jan.	124.37	118.07	113.40	108.54	113.03	129.55
Feb.	101.69	97.63	120.51	122.28	116.11	117.24
Mar.	111.51	110.54	122.49	118.18	108.34	116.44
Apr.	95.08	101.29	101.81	84.58	96.68	96.85
May	99.25	84.52	95.18	86.71	119.33	98.93
June	96.43	97.51	97.86	104.71	105.73	83.79
July	107.47	100.92	102.77	86.76	96.82	99.47
Aug.	120.62	117.87	116.99	150.64	109.69	125.47
Sept.	95.56	103.17	93.49	110.07	84.36	79.11
Oct.	106.78	94.54	81.11	97.58	100.74	101.88
Nov.	74.54	66.86	72.97	76.05	75.33	83.07
Dec.	81.06	94.61	105.62	79.92	84.81	97.27

4. Using the data on boot and shoe production given in problem G1, compute indexes of seasonal variation by the method of averaging the percentages of actual monthly values to their respective trend values. The linear equation of trend based on the years 1922–1940 is $y = 28.55 + .037x$ (origin at January, 1929). The required percentages of actual to trend data for the first six years of the period are given in the following table.

(a) Plot the seasonal indexes on an arithmetic grid.

Percentage Relation of Actual Boot and Shoe Production to Linear Trend Values

	1929	1930	1931	1932	1933	1934
Jan.	95.45	91.48	67.56	71.03	74.91	84.60
Feb.	96.92	89.22	81.31	86.76	86.86	99.09
Mar.	107.93	98.49	99.46	102.40	93.98	115.24
Apr.	102.51	99.62	101.15	86.50	90.77	111.46
May	101.60	84.08	96.15	74.90	108.17	110.12
June	97.84	81.91	93.96	78.35	114.22	92.18
July	105.00	82.55	96.46	67.88	110.47	91.58
Aug.	126.52	97.16	112.69	102.12	121.02	114.79
Sept.	120.73	100.14	105.21	112.29	101.96	90.70
Oct.	128.73	94.54	85.22	109.43	102.58	92.29
Nov.	95.85	63.13	62.13	83.11	77.20	76.57
Dec.	77.59	59.64	65.53	66.36	65.39	74.41

5. Using the data on boot and shoe production given in problem G1, compute indexes of seasonal variation by the method of averaging the percentages of actual monthly values to their centered twelve-month moving averages. The required percentages for the first six years of the period are given in the following table.

(a) Plot the seasonal indexes on an arithmetic grid.

Percentage Relation of Actual Boot and Shoe Production to 12-Month Moving Averages

	1929	1930	1931	1932	1933	1934
Jan.	93.58	92.47	77.66	84.18	79.44	85.91
Feb.	94.67	92.17	92.16	104.80	89.67	101.67
Mar.	104.78	103.96	111.63	123.91	96.65	119.14
Apr.	98.59	107.73	113.74	103.02	94.01	116.32
May	97.14	93.76	108.67	87.24	112.68	115.50
June	93.45	93.54	105.98	90.30	119.38	96.32
July	100.43	96.21	108.33	78.10	115.03	94.95
Aug.	121.58	115.05	126.06	117.30	124.86	118.46
Sept.	116.84	118.94	117.28	129.45	103.72	93.84
Oct.	125.22	112.13	95.49	126.41	102.51	95.86
Nov.	94.00	74.37	70.80	94.30	76.40	79.87
Dec.	77.16	69.44	75.99	72.88	65.26	78.14

6. The following table gives the monthly loadings of freight cars of Class I steam railways in the United States for the years 1929–1940.

Total Freight Car Loadings (Class I Steam Railways) in the United States, by months, 1929–1940 (thousand cars)

	1929	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939	1940	Av.
Jan.	4,519	4,246	3,491	2,267	1,924	2,183	2,170	2,433	2,714	2,257	2,302	2,558	2,755
Feb.	3,797	3,507	2,836	2,243	1,971	2,314	2,326	2,512	2,763	2,156	2,297	2,489	2,601
Mar.	3,838	3,516	2,940	2,826	2,355	3,068	3,015	2,415	2,986	2,223	2,390	3,123	2,891
Apr.	3,989	3,619	2,986	2,229	2,026	2,340	2,303	3,215	3,713	2,650	2,832	2,494	2,866
May	5,182	4,593	3,736	2,088	2,143	2,446	2,327	2,681	3,099	2,186	2,372	2,713	2,964
June	4,292	3,719	2,992	2,455	2,926	3,085	3,035	3,436	3,765	2,760	3,149	3,535	3,262
July	4,160	3,556	2,931	1,933	2,498	2,351	2,229	2,923	2,992	2,273	2,549	2,826	2,768
Aug.	5,601	4,672	3,747	2,626	3,205	3,073	3,102	2,955	3,101	2,392	2,689	3,718	3,406
Sept.	4,542	3,726	2,908	2,306	2,567	2,502	2,632	3,882	4,013	3,244	3,844	3,315	3,275
Oct.	5,752	4,751	3,813	2,534	2,632	2,535	2,882	3,278	3,157	2,843	3,375	3,269	3,402
Nov.	3,818	3,191	2,620	2,737	2,885	2,843	3,179	3,015	2,615	2,528	3,040	3,780	3,021
Dec.	3,338	2,782	2,273	1,936	2,088	2,106	2,319	3,365	2,753	2,947	3,262	2,718	2,657
Av.	4,402	3,823	3,108	2,348	2,435	2,570	2,627	3,009	3,139	2,538	2,842	3,466	3,026

Source: American Railway Association

(a) Calculate indexes of seasonal variation by the method of averaging the percentages of the original values to the average month of the year. The percentage calculations for the first six years of the period are given in the table below.

(b) Plot the seasonal indexes on an arithmetic grid.

Percentage Relation of Monthly Freight Car Loadings to Average Month of the Year

	1929	1930	1931	1932	1933	1934
Jan.	102.66	111.06	112.32	96.55	79.01	84.94
Feb.	86.26	91.73	91.25	95.53	80.94	90.04
Mar.	87.19	91.97	94.59	120.36	96.71	119.38
Apr.	90.62	94.66	96.07	94.93	83.20	91.05
May	117.72	120.14	120.21	88.93	88.01	95.18
June	97.50	97.28	96.27	104.56	120.16	120.04
July	94.50	93.02	94.31	82.33	102.59	91.48
Aug.	127.24	122.21	120.56	111.84	131.62	119.57
Sept.	103.18	97.46	93.56	98.21	105.42	97.35
Oct.	130.67	124.27	122.68	107.92	108.09	98.64
Nov.	86.73	83.47	84.30	116.57	118.48	110.62
Dec.	75.83	72.77	73.13	82.45	85.75	81.95

7. Using the data on freight car loadings given in problem F6, compute indexes of seasonal variation by the method of simple averages. For the purpose of adjusting the original averages of the twelve months, the linear least squares equation of trend, calculated from the original data for the years 1918–1937, is $y = 3604.70 - 5.2122x$ (origin at January, 1929).

(a) Plot the seasonal indexes on an arithmetic grid.

8. Using the data on freight car loadings given in problem F6, compute indexes of seasonal variation by the link-relative method. The link-relatives for the first six years of the period are given in the table below.

(a) Plot the seasonal indexes on an arithmetic grid.

Percentage Relation of Actual Freight Car Loadings to Preceding Month

	1929	1930	1931	1932	1933	1934
Jan.	128.49	127.20	125.49	99.74	99.38	104.55
Feb.	84.02	82.60	81.24	98.94	102.44	106.00
Mar.	101.08	100.26	103.67	125.99	119.48	132.58
Apr.	103.93	102.93	101.56	78.87	86.03	76.27
May	121.91	126.91	125.12	93.67	105.77	104.53
June	82.94	80.97	80.09	117.58	136.54	126.12
July	96.92	95.62	97.96	78.74	85.37	76.21
Aug.	134.64	131.38	127.84	135.85	128.30	130.71
Sept.	81.09	79.75	77.61	87.81	80.09	81.42
Oct.	126.64	127.51	131.12	109.89	102.53	101.32
Nov.	66.38	67.16	68.71	108.01	109.61	112.15
Dec.	87.43	87.18	86.76	70.73	72.37	74.08

9. Using the data on freight car loadings given in problem F6, compute indexes of seasonal variation by the method of averaging the percentages of actual monthly values to their respective trend values. The linear equation of trend, based on the years 1918–1937, is $y = 3604.70 - 5.2122x$ (origin at January, 1929).

The required percentages of actual to trend data for the first six years of the period are given in the table below.

(a) Plot the seasonal indexes on an arithmetic grid.

Percentage Relation of Actual Freight Car Loadings to Linear Trend Values

	1929	1930	1931	1932	1933	1934
Jan.	125.36	119.87	100.33	66.34	57.36	66.31
Feb.	105.49	99.15	81.63	65.74	58.85	70.40
Mar.	109.84	99.56	84.75	82.96	70.42	93.49
Apr.	111.14	102.62	86.20	65.53	60.68	71.42
May	144.59	130.43	108.01	61.48	64.28	74.78
June	119.94	105.77	86.64	72.40	87.91	94.46
July	116.42	101.28	85.00	57.09	75.17	72.10
Aug.	156.97	133.27	108.83	77.68	97.59	94.39
Sept.	127.48	106.44	84.59	68.32	77.49	76.98
Oct.	161.67	135.93	111.08	75.19	79.57	78.12
Nov.	107.47	91.43	76.44	81.34	87.36	87.75
Dec.	94.10	79.83	66.42	57.62	63.33	65.11

10. Using the data on freight car loadings given in problem F6, compute indexes of seasonal variation by the method of averaging the percentages of actual monthly values to their centered twelve-month moving averages. The required percentages for the first six years of the period are given in the table below.

(a) Plot the seasonal indexes on an arithmetic grid.

Percentage Relation of Actual Freight Car Loadings to 12-Month Moving Averages

	1929	1930	1931	1932	1933	1934
Jan.	102.60	101.68	101.32	85.28	83.35	83.86
Feb.	85.83	85.29	83.88	87.27	83.66	89.29
Mar.	86.49	87.05	88.87	113.12	99.54	118.76
Apr.	89.88	91.31	92.30	92.11	85.09	90.82
May	117.07	117.90	117.77	88.05	88.69	95.15
June	97.33	96.69	95.67	103.92	120.48	120.05
July	94.74	93.78	95.94	82.82	102.13	91.48
Aug.	128.24	125.18	125.76	113.76	129.71	119.58
Sept.	104.61	101.24	98.58	101.25	102.07	97.42
Oct.	133.36	130.87	130.87	112.65	102.90	98.85
Nov.	89.35	89.44	93.13	122.01	111.67	111.14
Dec.	79.01	79.44	83.49	85.47	80.23	82.56

11. Using the data on mail order and store sales of Montgomery Ward & Company given in problem H1, calculate indexes of seasonal variation by a method to be assigned by the instructor. Note that the percentages to moving-averages and to linear trend, for the years 1929–1934, are included in problem H1.

12. Using the data on sales of electric power to ultimate consumers given in problem H2, calculate indexes of seasonal variation by a method to be assigned

by the instructor. Note that the percentages to moving-averages and to trend, for the years 1929–1934 are included in problem H2.

13. From one of the time series given in monthly form in Table 15 of Appendix A, calculate indexes of seasonal variation by a method to be assigned by the instructor.

14. Inspection of the plotted data of the index of production of durable manufactures, constructed by the Federal Reserve Board, for the years 1926–1939 indicates distinctive seasonal patterns for the two periods 1926–1932 and 1934–1939. It is desired to calculate a separate set of seasonal indexes for each of the two periods.

Federal Reserve Board Index of Durable Manufactures
(1935–1939 average = 100)

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1926	109	115	119	120	120	116	112	119	117	117	109	100
1927	103	112	118	118	118	110	105	108	104	103	96	94
1928	101	110	116	120	119	118	116	122	124	125	118	115
1929	120	130	141	144	147	146	139	140	138	132	114	100
1930	105	116	118	119	116	108	93	92	89	83	76	69
1931	71	78	83	84	82	73	66	63	58	54	52	49
1932	48	49	46	46	46	42	36	34	37	38	37	36
1934	55	66	74	81	82	79	63	59	57	55	54	61
1935	72	81	84	85	81	80	78	82	81	90	97	96
1936	89	88	98	110	115	114	111	110	106	114	122	124
1937	120	124	133	140	143	132	131	133	118	114	96	80
1938	71	71	73	72	71	67	70	74	78	90	99	97
1939	93	94	98	98	95	100	98	99	111	131	136	139

Source: Federal Reserve Board

(a) From the following annual data, determine an appropriate equation of trend and calculate the trend value for each month of the two periods.

Annual Indexes of Production of Durable Manufactures
(1935–1939 = 100)

1919	84	1926	114	1933	54
1920	93	1927	107	1934	66
1921	53	1928	117	1935	84
1922	95	1929	133	1936	108
1923	104	1930	99	1937	122
1924	95	1931	68	1938	78
1925	108	1932	41	1939	108

(b) Correct the two series for the influence of trend by dividing each original value by the corresponding trend value.

(c) Plot the corrected series in the form of a tier chart (i.e., plot the percentages of actual to trend for each year directly under the preceding year's percentages).

(d) By the ratio-to-trend method, calculate seasonal indexes for each of the two periods.

H. ANALYSIS OF TIME SERIES; CYCLICAL VARIATION

Inasmuch as the segregation of cyclical fluctuations may be accomplished by several sequences of statistical operations, four procedures in common use are outlined. From these a selection may be made by the instructor.

Procedure A.

1. From the original data determine an appropriate trend value for each month of the series.

2. Remove trend from the original data by dividing each month's figure by the corresponding trend figure.

3. Calculate indexes of seasonal variation by one of the following methods:
   a. ratio-to-trend
   b. moving average
   c. link-relative
   d. simple averages
   e. percentage relation of each month to average month of the year.

4. Remove seasonal variation from the ratios-to-trend (obtained in step 2) by division (or subtraction).

5.* Run a 3, 5 or 7 month's moving average over the cyclic-irregular data (obtained in step 4) as an approximation of the cyclical variation.

Procedure B.

1. From the original data determine an appropriate trend value for each month of the series.

2. Remove trend from the original data by dividing each month's figure by the corresponding trend figure.

3. Calculate indexes of seasonal variation by one of the following methods:
   a. ratio-to-trend
   b. moving average
   c. link-relative
   d. simple averages
   e. percentage relation of each month to average month of the year.

4. Multiply each month's trend value by the corresponding seasonal index to obtain the "expected" figure resulting from trend and seasonal forces in combination.

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* Inasmuch as the cyclical and random movements are so closely interwoven and due to the absence of a satisfactory and simple means of separating the two movements, the final step in the procedure as outlined is to be considered, merely, as an attempt to eliminate *some* of the effects of the random movements. The short term moving average or other smoothing procedure is to be treated, accordingly, as an optional part of the technique.

5. Divide each month's original figure by the "expected" figure to secure the cyclic-irregular variations.

6.* Run a 3, 5 or 7 month's moving average over the cyclic-irregular data (obtained in step 5) as an approximation of the cyclical variation.

Procedure C.

1. From the original data determine an appropriate trend value for each month of the series.

2. Remove trend from the original data by dividing each month's figure by the corresponding trend figure.

3. Calculate indexes of seasonal variation by one of the following methods:
   a. ratio-to-trend
   b. moving average
   c. link-relative
   d. simple averages
   e. percentage relation of each month to average month of the year.

4. Multiply each month's trend value by the corresponding seasonal index to obtain the "expected" figure resulting from trend and seasonal forces in combination.

5. Subtract the "expected" figures from the original data to secure the cyclic-irregular variations (in original units).

6. Divide the cyclic-irregular data of step 5 by the "expected" data of step 4 to secure the cyclic-irregular variations as percentage departures from the trend and seasonal variations in combination.

7.* Run a 3, 5 or 7 month's moving average over the cyclic-irregular data (obtained in step 6) as an approximation of the cyclical variation.

Procedure D.

1. Calculate indexes of seasonal variation by one of the following methods:
   a. ratio-to-trend
   b. moving average
   c. link-relative
   d. simple averages
   e. percentage relation of each month to average month of the year.

2. Remove seasonal variation from the original data by dividing each month's figure by the corresponding seasonal index.

3. Determine an appropriate trend value for each month of the original data.

4. Divide the seasonally adjusted data (obtained in step 2) by the corresponding trend data to secure the cyclic-irregular variations.

5.* Run a 3, 5 or 7 month's moving average over the cyclic-irregular variations as an approximation of the cyclical variation.

* See footnote on page 51.

1. The following table gives the mail order and store sales of Montgomery Ward & Company by months for the years 1929–1940.

Mail Order and Store Sales of Montgomery Ward & Co. for the Years 1929–1940
(millions of dollars)

	1929	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939	1940
Jan.	18.13	18.33	16.62	12.03	10.10	14.78	17.42	18.51	22.58	21.84	24.77	29.98
Feb.	18.67	18.41	15.25	11.96	10.13	15.42	17.91	17.86	22.16	21.77	24.96	30.53
Mar.	22.62	20.63	17.60	14.05	11.26	18.31	22.78	24.85	34.93	30.80	35.73	38.84
Apr.	21.57	23.78	20.56	16.17	15.67	20.87	25.57	30.40	40.10	37.06	41.60	45.86
May	19.88	25.05	18.55	14.17	15.25	20.94	22.92	30.30	39.14	36.15	42.32	45.91
June	21.95	23.99	19.22	14.84	16.10	19.27	23.82	30.33	37.06	35.75	41.30	43.10
July	19.81	18.67	15.32	11.80	13.64	15.89	20.29	25.64	30.44	29.08	33.45	37.21
Aug.	24.96	20.97	16.41	12.99	15.39	18.92	22.85	27.42	29.68	32.85	39.00	42.69
Sept.	26.13	21.33	17.51	14.64	16.58	23.09	25.17	33.36	37.46	38.56	44.74	45.97
Oct.	32.18	30.09	22.03	19.81	22.78	29.70	35.90	45.46	48.83	46.67	54.95	56.94
Nov.	29.85	22.40	18.40	16.55	20.97	26.90	30.91	36.98	39.55	42.30	47.76	54.61
Dec.	35.80	28.67	21.90	21.06	24.86	34.68	39.48	51.79	51.36	57.09	66.02	70.85
Ave.	24.29	22.69	18.28	15.01	16.06	21.56	25.42	31.07	36.11	35.82	41.38	45.21

Source: Survey of Current Business

(a) Following Procedure A, B, C or D above, perform a complete time series analysis on the above sales data. The percentages of original data to 12-months, centered, moving averages and to the linear trend values (from the equation of trend $y = 22.8196 + .1386x$, origin at July 1, 1931) for the first six years of the period are given in the following table.

(b) How do you interpret the final data? In what manner does the selection of the time period, the type of trend equation and the method of calculating the seasonal indexes affect the final results?

Relation of Montgomery Ward & Co. Sales to Trend and Trend-Cycle
for the Years 1929–1940

	Trend Value for the Month	Percentage of Actual to Trend	12-month's Moving Average of Actual (centered)	Percentage of Actual to 12-month's Average
1929				
Jan.	18.73	96.80	21.79	83.20
Feb.	18.87	98.94	22.37	83.46
Mar.	19.01	118.99	22.92	98.69
Apr.	19.15	112.64	23.38	92.26
May	19.29	103.06	23.79	83.56
June	19.42	113.03	24.14	90.93
July	19.56	101.28	24.30	81.52
Aug.	19.70	126.70	24.30	102.72
Sept.	19.84	131.70	24.21	107.93
Oct.	19.98	161.06	24.22	132.87
Nov.	20.12	148.36	24.52	121.74
Dec.	20.26	176.70	24.82	144.24
1930				
Jan.	20.39	89.90	24.86	73.73
Feb.	20.53	89.67	24.65	74.69

Relation of Montgomery Ward & Co. Sales to Trend and Trend-Cycle for the Years 1929–1940—Continued

	Trend Value for the Month	Percentage of Actual to Trend	12-month's Moving Average of Actual (centered)	Percentage of Actual to 12-month's Average
Mar.	20.67	99.81	24.28	84.97
Apr.	20.81	114.27	24.00	99.08
May	20.95	119.57	23.60	106.14
June	21.09	113.75	22.99	104.35
July	21.23	87.94	22.62	82.54
Aug.	21.36	98.17	22.42	93.53
Sept.	21.50	99.21	22.16	96.25
Oct.	21.64	139.05	21.90	137.40
Nov.	21.78	102.85	21.50	104.19
Dec.	21.92	130.79	21.03	136.33
1931				
Jan.	22.06	75.34	20.69	80.33
Feb.	22.20	68.69	20.36	74.90
Mar.	22.33	78.82	20.01	87.96
Apr.	22.47	91.50	19.51	105.38
May	22.51	82.41	19.01	97.58
June	22.75	84.48	18.36	103.56
July	22.89	66.93	18.09	84.69
Aug.	23.03	71.25	17.76	92.40
Sept.	23.17	75.57	17.48	100.17
Oct.	23.30	94.55	17.15	128.45
Nov.	23.44	78.50	16.78	109.65
Dec.	23.58	92.88	16.41	133.46
1932				
Jan.	23.72	50.72	16.09	74.77
Feb.	23.86	50.13	15.80	75.70
Mar.	24.00	58.54	15.53	90.47
Apr.	24.14	66.98	15.32	105.55
May	24.27	58.38	15.15	93.53
June	24.41	60.79	15.04	98.67
July	24.55	48.07	14.93	79.04
Aug.	24.69	52.61	14.77	87.95
Sept.	24.83	58.96	14.58	100.41
Oct.	24.97	79.34	14.44	137.19
Nov.	25.11	65.91	14.46	114.45
Dec.	25.25	83.41	14.56	144.65
1933				
Jan.	25.38	39.80	14.69	68.75
Feb.	25.52	39.69	14.87	68.12
Mar.	25.66	43.88	15.05	74.82
Apr.	25.80	60.74	15.25	102.75
May	25.94	58.79	15.56	98.01
June	26.08	61.73	15.90	101.26

Relation of Montgomery Ward & Co. Sales to Trend and Trend-Cycle for the Years 1929–1940—Continued

	Trend Value for the Month	Percentage of Actual to Trend	12-month's Moving Average of Actual (centered)	Percentage of Actual to 12-month's Average
July	26.22	52.02	16.26	83.89
Aug.	26.35	58.41	16.67	92.32
Sept.	26.49	62.59	17.19	96.45
Oct.	26.63	85.54	17.70	128.70
Nov.	26.77	78.33	18.15	115.54
Dec.	26.91	92.38	18.52	134.23
1934				
Jan.	27.05	54.64	18.74	78.87
Feb.	27.19	56.71	18.99	81.20
Mar.	27.32	67.02	19.40	94.38
Apr.	27.46	76.00	19.96	104.56
May	27.60	75.87	20.50	102.15
June	27.74	69.47	21.15	91.11
July	27.88	56.99	21.67	73.33
Aug.	28.02	67.52	21.89	86.43
Sept.	28.16	82.00	22.18	104.10
Oct.	28.29	104.98	22.56	131.65
Nov.	28.43	94.62	22.84	117.78
Dec.	28.57	121.39	23.11	150.06

2. The following table gives the sales to ultimate consumers of electric power in the United States for the years 1929–1940, by months.

Sales to Ultimate Consumers of Electric Power for the Years 1929–1940 (million kilowatt hours)

	1929	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939	1940
Jan.	6.28	6.72	6.33	5.89	5.37	5.91	6.47	7.24	8.33	7.93	8.81	10.07
Feb.	6.03	6.28	5.94	5.62	5.03	5.77	6.19	6.97	7.95	7.43	8.32	9.50
Mar.	5.98	6.22	5.95	5.48	4.88	5.80	6.08	6.84	8.22	7.47	8.40	9.33
Apr.	6.15	6.25	6.09	5.31	4.99	5.84	6.23	7.06	8.25	7.36	8.24	9.27
May	6.21	6.25	6.02	5.10	5.24	5.92	6.15	7.05	8.12	7.23	8.28	9.37
June	6.11	6.15	5.92	5.02	5.60	5.88	6.15	7.28	8.26	7.44	8.58	9.47
July	6.09	6.03	5.91	4.97	5.76	5.81	6.27	7.55	8.36	7.56	8.58	9.61
Aug.	6.45	6.12	5.97	5.05	5.87	5.98	6.61	7.76	8.62	8.09	8.95	10.10
Sept.	6.39	6.17	5.89	5.24	5.83	5.77	6.64	7.86	8.64	8.19	9.27	10.06
Oct.	6.56	6.30	5.94	5.33	5.78	5.99	6.82	8.05	8.47	8.34	9.64	10.40
Nov.	6.58	6.22	5.83	5.42	5.72	5.99	6.93	8.01	8.19	8.48	9.68	10.58
Dec.	6.46	6.20	5.89	5.35	5.69	6.13	7.03	8.38	8.05	8.78	9.76	10.90
Ave.	6.28	6.24	5.97	5.31	5.48	5.90	6.46	7.50	8.29	7.86	8.88	9.89

Source: Survey of Current Business

(a) Following Procedure A, B, C or D outlined above, perform a complete time series analysis on the above series. The percentages of original data to 12-month, centered, moving averages and to the linear trend values (from the equation of trend $y = 5.292 + .025x$, origin at January 1, 1929) for the first six years of the period are given in the following table.

(b) Interpret specifically the final data. What possible "errors" are to be considered in evaluating the accuracy of the analysis?

Relation of Electric Power Sales to Trend and to Trend-Cycle for the Years 1929–1934

	Trend Value for the Month	Percentage of Actual to Trend	12-month's Moving Average of Actual (centered)	Percentage of Actual to 12-month's Average
1929				
Jan.	5.292	118.67	5.983	104.96
Feb.	5.317	113.41	6.048	99.70
Mar.	5.342	111.94	6.112	97.84
Apr.	5.367	114.59	6.164	99.77
May	5.392	115.17	6.207	100.05
June	5.417	112.79	6.251	97.34
July	5.442	111.91	6.292	96.79
Aug.	5.467	117.98	6.321	102.04
Sept.	5.492	116.35	6.342	100.76
Oct.	5.517	118.91	6.356	103.21
Nov.	5.542	118.73	6.362	103.43
Dec.	5.567	116.04	6.365	101.49
1930				
Jan.	5.592	120.17	6.364	105.59
Feb.	5.617	111.80	6.348	98.93
Mar.	5.642	110.24	6.325	98.34
Apr.	5.667	110.29	6.305	99.13
May	5.692	109.80	6.279	99.54
June	5.717	107.57	6.253	98.35
July	5.742	105.02	6.226	96.85
Aug.	5.767	106.12	6.196	98.77
Sept.	5.792	106.53	6.170	100.00
Oct.	5.817	108.30	6.152	102.41
Nov.	5.842	106.47	6.136	101.37
Dec.	5.867	105.68	6.117	101.36
1931				
Jan.	5.892	107.43	6.102	103.74
Feb.	5.917	100.39	6.091	97.52
Mar.	5.942	100.13	6.073	97.97
Apr.	5.967	102.06	6.047	100.71
May	5.992	100.47	6.015	100.08
June	6.017	98.39	5.986	98.90

Relation of Electric Power Sales to Trend and to Trend-Cycle for the Years 1929–1934—Continued

	Trend Value for the Month	Percentage of Actual to Trend	12-month's Moving Average of Actual (centered)	Percentage of Actual to 12-month's Average
July	6.042	97.82	5.955	99.24
Aug.	6.067	98.40	5.923	100.79
Sept.	6.092	96.68	5.890	100.00
Oct.	6.117	97.11	5.838	101.75
Nov.	6.142	94.92	5.767	101.09
Dec.	6.167	95.51	5.692	103.48
1932				
Jan.	6.192	95.12	5.615	104.90
Feb.	6.217	90.40	5.537	101.50
Mar.	6.242	87.79	5.472	100.15
Apr.	6.267	84.73	5.420	97.97
May	6.292	81.06	5.377	94.85
June	6.317	79.47	5.337	94.06
July	6.342	78.37	5.293	93.90
Aug.	6.367	79.32	5.247	96.25
Sept.	6.392	81.98	5.197	100.83
Oct.	6.417	83.06	5.159	103.31
Nov.	6.442	84.14	5.152	105.20
Dec.	6.467	82.73	5.182	103.24
1933				
Jan.	6.492	82.72	5.239	102.50
Feb.	6.517	77.18	5.306	94.80
Mar.	6.542	74.59	5.365	90.96
Apr.	6.567	75.99	5.408	92.27
May	6.592	79.49	5.439	96.34
June	6.617	84.63	5.466	102.45
July	6.642	86.72	5.502	104.69
Aug.	6.667	88.05	5.556	105.65
Sept.	6.692	87.12	5.625	103.64
Oct.	6.717	86.05	5.698	101.42
Nov.	6.742	84.84	5.762	99.27
Dec.	6.767	84.08	5.803	98.07
1934				
Jan.	6.792	87.01	5.816	101.62
Feb.	6.817	84.64	5.823	99.09
Mar.	6.842	84.77	5.825	99.57
Apr.	6.867	85.04	5.831	100.15
May	6.892	85.90	5.851	101.18
June	6.917	85.01	5.881	99.98
July	6.942	83.69	5.922	98.11

Relation of Electric Power Sales to Trend and to Trend-Cycle for the Years 1929–1934—Continued

	Trend Value for the Month	Percentage of Actual to Trend	12-month's Moving Average of Actual (centered)	Percentage of Actual to 12-month's Average
Aug.	6.967	85.83	5.963	100.29
Sept.	6.992	82.52	5.992	96.30
Oct.	7.017	85.36	6.020	99.50
Nov.	7.042	85.06	6.046	99.07
Dec.	7.067	86.74	6.072	100.96

3. Using the data on boot and shoe production from problem G1, perform a complete time series analysis by Procedure A, B, C or D as assigned by the instructor.

4. Using the data on freight car loadings from problem G6, perform a complete time series analysis by Procedure A, B, C or D as assigned by the instructor.

5. Using the data in Table 15 of Appendix A giving Portland cement production, perform a complete time-series analysis by Procedure A, B, C or D as assigned by the instructor.

6. Using the data in Table 15 of Appendix A on meat consumption, perform a complete time-series analysis by Procedure A, B, C or D as assigned by the instructor.

7. Using the data in Table 15 of Appendix A on cotton consumption, perform a complete time-series analysis by Procedure A, B, C or D as assigned by the instructor.

8. Using the data in Table 15 of Appendix A on bank debits, perform a complete time-series analysis by Procedure A, B, C or D as assigned by the instructor.

9. What do the final data in problem 3 (or 4, 5, 6, 7 or 8) measure specifically? Of what importance is each of the following in regard to the interpretation?
   (a) the period of years included in the analysis
   (b) the choice of trend equation
   (c) the type of seasonal indexes calculated
   (d) the method of adjusting for trend and seasonal influences

I. INDEXES OF PHYSICAL VOLUME

1. The following table gives the average monthly production and wholesale prices of fuels and by-products for the years 1923–1940.

Production and Wholesale Prices of Fuels and By-Products in the United States for the Years 1923–1940 (monthly averages)

	Anthracite Coal Production	Dollars per Short Ton	Bituminous Coal Production	Dollars per Short Ton	Coke Production	Dollars per Short Ton	Crude Petroleum Production	Dollars per Barrel
1923	7.78	12.40	47.05	4.83	4.80	5.55	61.0	1.44
1924	7.33	12.42	40.31	4.21	3.75	3.53	59.5	1.45
1925	5.15	12.31	43.34	4.12	4.36	4.05	63.6	1.67
1926	7.04	12.29	47.78	4.31	4.82	4.11	64.2	1.88
1927	6.68	11.87	43.15	4.26	4.35	3.20	75.1	1.29
1928	6.28	11.61	41.73	4.03	4.52	2.79	75.1	1.20
1929	6.15	11.51	44.58	3.95	5.14	2.78	83.9	1.23
1930	5.78	11.35	38.96	3.91	4.06	2.57	74.8	1.11
1931	4.97	11.40	31.84	3.74	2.96	2.46	70.9	.57
1932	4.16	10.88	25.81	3.64	1.96	2.12	65.4	.81
1933	4.13	10.06	27.80	3.67	2.43	2.50	75.5	.57
1934	4.76	9.64	29.93	4.13	2.76	3.65	75.7	.94
1935	4.35	9.59	31.03	4.24	3.05	3.57	83.1	.94
1936	4.55	9.74	36.59	4.27	3.97	3.68	91.6	1.04
1937	4.32	9.37	37.13	4.29	4.47	4.38	106.6	1.16
1938	3.84	9.45	29.05	4.33	2.86	3.97	101.2	1.12
1939	4.23	9.14	32.44	4.31	3.84	4.09	105.4	.95
1940	4.17	9.55	37.77	4.31	4.86	4.56	112.7	.96

Source: Survey of Current Business

Prices: anthracite coal, composite, chestnut; bituminous coal, mine run, composite; coke, Connellsville, beehive; crude petroleum, Kansas-Oklahoma, at wells.

Production: coal and coke data in millions of short tons; petroleum data in millions of barrels.

(a) Construct an index of fuel and by-products production for the years 1923–1940 by the simple average of relatives method,
  (1) using the year 1926 as the base year;
  (2) *alternative assignment:* using the average month of 1923–1925 as the base.
(b) Interpret the index for 1932; for 1940.
(c) Is each series given an equal weight (i.e., a weight of 1) in the calculations?
(d) What are the limitations of this type of index?

2. The table below gives the volume of production and wholesale prices of certain dairy products by months in the United States for the years 1923–1940.

Volume of Production and Wholesale Prices of Selected Dairy Products in the United States for the Years 1923–1940 (monthly averages)

	Butter		Cheese		Condensed Milk		Evaporated Milk	
	Production	Price per lb.	Production	Price per lb.	Production	Price per case	Production	Price per case
1923	103.5	$.47	32.9	$.25	394.5	$6.24	2,399.5	$4.27
1924	113.0	.43	34.5	.21	375.6	5.98	2,279.2	3.76
1925	113.5	.45	37.0	.24	376.9	5.91	2,303.6	3.93
1926	121.0	.44	35.6	.23	310.0	5.86	2,219.3	3.98
1927	124.7	.47	33.9	.26	323.4	5.87	2,440.3	4.14
1928	123.9	.47	36.5	.25	278.7	6.01	2,561.4	4.01
1929	133.1	.45	40.3	.24	292.8	6.14	2,872.9	3.87
1930	132.9	.37	41.7	.20	245.5	6.07	2,776.1	3.49
1931	139.0	.28	41.0	.15	196.9	5.32	2,737.5	3.21
1932	141.2	.21	40.3	.13	141.8	4.72	3,008.8	2.53
1933	146.9	.22	45.3	.13	109.4	4.70	3,288.7	2.55
1934	141.2	.26	48.3	.14	123.7	4.85	3,278.9	2.70
1935	136.0	.30	51.7	.17	108.8	4.85	3,522.8	2.92
1936	135.8	.33	53.5	.18	97.4	4.85	3,915.2	3.26
1937	135.3	.34	54.1	.18	98.4	4.87	3,644.7	3.21
1938	148.8	.28	60.4	.15	86.5	5.00	4,031.0	3.20
1939	146.8	.26	58.4	.16	71.8	5.00	4,370.7	2.95
1940	150.7	.30	64.1	.17	126.0	5.00	4,736.7	3.09

Source: Survey of Current Business

Production: butter, creamery, factory in millions of pounds; cheese, total factory in millions of pounds; condensed milk, sweetened, case goods in thousands of cases (42 pounds per case); evaporated milk, unsweetened, case goods in thousands of cases (43.5 pounds per case).

Prices: butter, creamery, 92 score (N. Y.); cheese, No. 1, American (N. Y.); condensed and evaporated milk (N. Y.).

(a) Construct an index of production of dairy products for the years 1923–1940 by the simple average of relatives method,
  (1) using the year 1926 as the base year;
  (2) *alternative assignment:* using the average month of 1923–1925 as the base.
(b) Give a specific interpretation of the indexes for 1923 and 1939.
(c) Which of the series actually is most dominant in the calculations for the same two years?

3. The table below gives the volume of imports and wholesale prices of certain tropical food products by months for the years 1923–1939.

Volume of Imports and Wholesale Prices of Selected Tropical Food Products for the Years 1923–1939 (monthly averages in thousand long tons)

	Cocoa		Coffee		Raw Sugar		Refined Sugar		Tea	
	Volume of imp.	Dollars per lb.	Volume of imp.	Dollars per lb.	Volume of imp.	Dollars per lb.	Volume of imp.	Dollars per lb.	Volume of imp.	Dollars per lb.
1923	15.4	.074	52.4	.115	282	.070	3.9	.084	3.9	.310
1924	14.1	.075	52.9	.168	309	.060	1.3	.074	3.5	.317
1925	14.2	.097	47.7	.203	328	.043	1.5	.055	3.8	.350
1926	15.9	.116	55.6	.182	331	.043	6.1	.055	3.6	.355
1927	15.8	.160	53.3	.148	306	.047	9.0	.058	3.3	.342
1928	14.1	.129	54.2	.165	274	.042	16.3	.056	3.3	.321
1929	18.9	.104	55.2	.157	306	.038	20.4	.051	3.3	.318
1930	13.9	.080	59.5	.087	268	.034	21.5	.047	3.2	.272
1931	15.5	.052	64.8	.062	214	.033	28.0	.044	3.2	.224
1932	17.8	.044	55.9	.081	181	.032	32.8	.040	3.5	.185
1933	17.6	.044	59.0	.078	180	.032	30.8	.043	3.6	.175
1934	16.4	.052	56.7	.098	190	.030	32.6	.044	2.8	.204
1935	22.6	.050	65.3	.072	188	.032	31.6	.049	3.2	.269
1936	23.5	.069	64.7	.075	187	.036	33.6	.048	3.1	.275
1937	23.0	.084	63.1	.088	205	.035	32.5	.047	3.5	.276
1938	16.9	.053	74.0	.053	190	.029	31.6	.045	3.0	.280
1939	24.7	.049	75.0	.052	186	.030	30.1	.046	3.6	.280*

* Six-month's average.

Source: Survey of Current Business

(a) Construct an index of the volume of imports of tropical food products for the years 1923–1939 by the simple average of relatives method,
   (1) using the year 1926 as the base year;
   (2) *alternative assignment:* using the average month of 1923–1925 as the base.

4. (a) Using the data of problem 11, construct an index of fuel and by-products production for the years 1923–1940 by the method of weighted aggregates expressed as relatives,
   (1) using the year 1926 as the base with base year price weights.
   (2) *alternative assignment:* using the average month of 1923–1925 as the base with base month price weights.
(b) How do you interpret the index for 1939?
(c) Calculate the index for 1939 using the simple average of relatives method and compare it with the 1939 index interpreted in (b).

5. (a) Using the data of problem 12 construct an index of dairy products production for the years 1923–1940 by the method of weighted aggregates expressed as relatives,
   (1) using the year 1926 as the base with base year price weights;
   (2) *alternative assignment:* using the average month of 1923–1925 as the base with base month price weights.
(b) Interpret specifically the indexes for 1932 and 1940.

(c) How and why do these indexes differ from the results obtained in problem I2(a)?

6. Using the data of problem I3, construct an index of the volume of imports of tropical food products for the years 1923–1939 by the method of weighted aggregates expressed as relatives,

(1) using the year 1926 as the base with base year price weights;
(2) *alternative assignment:* using the average month of 1923–1925 as the base with base month price weights.

7. (a) Using the data of problem I1, construct an index of fuel and by-products production for the year 1930–1940 using the "ideal" formula,

(1) using the year 1926 as the base year;
(2) *alternative assignment:* using the average month of the period 1935–1939 as the base.

(b) Interpret the indexes for 1935 and 1937.
(c) What are the especial merits of this type of index? How is the year-to-year comparability affected by the changing weights?

8. Using the data of problem I2, construct an index of dairy products production for the years 1930–1940 using the "ideal" formula,

(a) using the year 1926 as the base year;
(b) *alternative assignment:* using the average month of the period 1935–1939 as the base.

9. Using the data of problem I3, construct an index of the volume of tropical food imports for the years 1930–1939 using the "ideal" formula,

(a) using the year 1926 as the base year;
(b) *alternative assignment:* using the average month of the period 1935–1939 as the base.

10. The table below gives annual data for the years 1920–1940 for 17 economic series commonly used in the construction of indexes of production and of business activity for the United States.

	Pig Iron Production (mil. short tons)	Steel Ingot Production (mil. short tons)	Bit. Coal Production (mil. short tons)	Coke Production (mil. short tons)	Cement Production (mil. bbls.)	Petroleum Production (mil. bbls.)	Auto Production* (thous. cars)	Shoe Production* (mil. pairs)	Textile Mfg.* (index, 1935–39 = 100)
1920	40.8	46.1	568.7	51.4	99.7	442.9	185.6	—	66
1921	18.5	21.6	415.9	25.3	98.3	472.2	134.7	—	69
1922	30.1	38.9	422.3	37.1	113.9	557.5	212.0	27.0	78
1923	44.9	49.0	564.6	57.0	137.4	732.4	336.2	29.8	83
1924	34.8	41.4	483.7	44.3	148.9	713.9	300.2	26.1	72
1925	40.8	49.7	520.1	51.3	161.2	763.7	355.5	27.0	84
1926	43.8	52.9	573.4	56.9	164.1	770.9	358.4	27.0	84
1927	40.6	49.3	517.4	51.1	171.9	901.1	283.4	28.6	92
1928	42.4	56.6	500.8	52.8	176.0	1,007.3	363.2	28.6	87
1929	47.4	61.7	535.0	59.9	170.2	898.0	446.5	30.1	94
1930	35.2	44.6	467.5	48.0	160.9	851.1	279.7	27.4	74
1931	20.5	28.6	382.1	33.5	124.6	785.2	199.1	29.4	79

1932	9.7	15.1	309.7	21.8	76.5	905.7	114.2	26.1	71
1933	14.8	25.7	333.6	27.6	63.4	908.1	160.0	29.2	88
1934	17.8	29.2	359.4	31.8	77.7	996.6	229.4	29.8	76
1935	23.5	38.2	372.4	35.1	76.5	1,099.7	328.9	32.0	93
1936	34.3	53.5	439.1	46.3	112.4	1,279.2	371.2	34.6	104
1937	41.0	56.6	445.5	52.4	116.5	1,213.3	400.7	34.3	106
1938	21.0	31.8	348.6	32.7	105.6	1,214.4	207.4	32.5	85
1939	35.3	52.8	389.5	44.4	121.8	1,264.3	298.1	35.3	112
1940	47.0	67.0	453.2	56.8	130.3	1,361.9	372.4	33.2	115

	Petroleum Refining (index* 1935–39 = 100)	Mfg. Employ't (index* 1923–25 = 100)	Contr's Aw'd* (index 1923–25 = 100)	Freight Car Load's* (mil. cars)	Dept. Store Sales* (index, 1923–25 = 100)	Exports from U. S.* (mil. dols)	Vol. of Stock Sales* (mil. shares)	Bank Debits* (outside N. Y. C.) (bil. dols)
1920	34	107	63	3.76	94	686	18.7	40.3
1921	34	82	56	3.28	87	374	14.2	33.3
1922	40	91	79	3.60	88	319	21.7	36.6
1923	46	104	84	4.15	98	347	19.8	38.6
1924	53	96	94	4.05	99	383	23.5	41.0
1925	62	100	122	4.27	103	409	37.7	47.5
1926	68	102	129	4.43	106	401	37.4	50.7
1927	72	100	129	4.30	107	405	48.1	56.2
1928	81	100	135	4.30	108	427	76.7	67.2
1929	90	106	117	4.40	111	437	93.8	77.9
1930	85	92	92	3.82	102	320	67.6	55.2
1931	80	78	63	3.10	92	202	48.1	40.1
1932	72	66	28	2.35	69	134	35.4	26.9
1933	75	73	25	2.44	67	140	54.6	25.7
1934	79	86	32	2.57	75	178	27.0	27.7
1935	86	91	37	2.63	79	190	31.8	31.2
1936	95	99	55	3.01	88	205	41.3	35.7
1937	106	109	59	3.14	92	279	34.1	36.1
1938	103	91	64	2.54	85	258	24.8	31.1
1939	111	100	72	2.84	90	265	21.8	32.5
1940	113	108	81	3.03	94	335	17.3	34.0

* Monthly averages.

Source: Survey of Current Business

(a) Construct an index of manufacturing for a period to be selected by the instructor using the following series and weights.

Series	Weight
Pig iron production	15
Steel ingot production	30
Automobile production	15
Petroleum refining	10
Boot and shoe production	10
Textile manufacturing	20
	100

The procedure involves the following steps:

(1) Fit an appropriate line of trend to each of the constituent series.

(2) Express the actual items of each series as percentages of their corresponding trend values.

(3) For each series, calculate the standard deviation of the deviations from trend.

(4) Divide the deviations from trend for each series by the respective standard deviations.

(5) Calculate for each year a weighted average of the deviations in standard deviation units.

(6) Multiply each year's index (as determined in step 5) by the weighted average standard deviation to secure the final index for the year as a percentage deviation from trend.

(b) Plot the index series on arithmetic paper with appropriate title and scales.

(c) Interpret the indexes for 1932 and 1940.

(d) What limitations to this index series may be pointed out?

11. (a) From data in the foregoing table, construct an index of manufacturing and trade for a period designated by the instructor, using the following components and weights:

Series	Weights
Steel ingot production	25
Automobile production	20
Manufacturing employment	25
Department store sales	20
U. S. exports	10
	100

Follow the procedure outlined in problem I10, deflating the department store sales data and the series on United States exports before fitting lines of trend.

(b) Plot the index series on arithmetic graph paper with appropriate title and scales.

(c) How do you interpret the calculated indexes?

(d) State the reasons for each of the steps in the index construction and point out the merits and limitations of the final series.

12. From data in the foregoing table, construct an index of business activity for a period designated by the instructor, using the following components and weights:

Series	Weights
Steel ingot production	25
Automobile production	15
Manufacturing employment	20
Freight car loadings	15
Department store sales	15
Bank debits (outside N. Y. C.)	10
	100

Follow the procedure outlined in problem I10, deflating department store sales data and the bank debits data before fitting lines of trend.

13. From data in the foregoing table, construct an index of business activity for a period designated by the instructor, using the following components and weights:

Series	Weights
Steel ingot production	30
Manufacturing employment	10
Textile manufacturing	15
Construction contracts awarded	15
Bank debits (outside N. Y. C.)	20
Stock sales (N. Y. C. Stock Exchange)	10
	100

J. SIMPLE CORRELATION; UNGROUPED DATA

1. The following data define the circulation per issue and the advertising rate per square inch for 18 United States magazines. The relationship between advertising rates and circulation is to be determined.

Magazine	"A. B. C." Circulation per Issue (in millions)	Advertising Rate per Square Inch
Vogue	.22	$12.60
House and Garden	.15	14.97
Vogue Pattern	.11	.53
American Magazine	2.18	70.13
Collier's	2.62	57.16
Woman's Home Companion	3.03	75.20
Country Home	1.65	40.46
Saturday Evening Post	3.10	70.36
Ladies' Home Journal	3.04	74.76
Country Gentleman	1.84	46.61
Esquire	.57	31.31
McCall's	2.82	68.60
Redbook	1.16	37.97
Blue Book	.10	2.67
McCall Fashion	.17	13.19
Time	.77	34.40
Fortune	.14	14.34
Life	1.89	50.13

Source: Poor's Industry and Investment Surveys

(a) Construct a scatter diagram using magazine circulation as the independent and advertising rate as the dependent variables.
(b) Calculate the linear least squares equation of regression and plot on the scatter diagram.
(c) Calculate the standard error of estimate of Y by taking the square root of the mean of the sum of the squared differences between the computed advertising rates and the original figures.
(d) Calculate the coefficient of correlation (r) from the formula

$$r^2 = 1 - \frac{S_y^2}{\sigma_y^2}.$$

2. Using the data of problem J1
(a) Calculate the linear least squares equation of regression and plot on a scatter diagram prepared from the original data.
(b) Calculate the standard error of estimate of Y using the formula

$$S_y^2 = \frac{\Sigma Y^2 - a\Sigma Y - b\Sigma XY}{N}.$$

(c) Calculate the coefficient of correlation (r) using the least squares formula.

3. Using the data of problem J1

(a) Calculate the coefficient of correlation (r) from the product moment formula.

(b) Calculate the linear least squares equation of regression $Y=a+bX$ from the product moment formula

$$Y-\overline{Y}=r\frac{\sigma_y}{\sigma_x}(X-\overline{X}).$$

($\overline{Y}$and $\overline{X}$ are the means of the two series.)

(c) Calculate the standard error of estimate of Y from the product moment formula

$$S_y=\sigma_y\sqrt{1-r^2}.$$

4. (a) Calculate and interpret the standard error of r, using the data of the preceding three problems.

$$\sigma_r=\frac{1-r^2}{\sqrt{N-1}}\cdot$$

(b) Determine whether the coefficient of correlation is significantly different from zero.

(c) Transform r to z, using Table IV of Appendix B, and indicate the possible ranges of error due to chance, giving the appropriate probabilities involved.

5. Table 16 of Appendix A gives five measures of debt-paying capacity for the forty-eight states of the Union. Using retail sales as the dependent variable and per cent of population making federal income tax returns as the independent variable, follow the instructions of problems J1 (or J2 or J3) and J4 above.

6. Table 17 of Appendix A gives the total taxes paid and the total assets for 100 companies having capitalizations under seventy million dollars. Using total assets as the independent variable and total taxes paid as the dependent variable, follow the instructions of problems J1 (or J2 or J3) and J4 above.

7. Data are given in Table 18 of Appendix A showing the yield at market price of 39 telephone, gas and electric preferred stocks and the number of times dividends and prior requirements were earned.

(a) Using the least squares formulae, calculate the appropriate curvilinear least squares equation of regression and plot on a scatter diagram of the material. Yields should be used as the dependent series and the number of times dividends were earned as the independent series.

(b) Determine the standard error of estimate of Y, the index of correlation, and the standard error of the correlation index (making appropriate correction for the number of constants used in the regression equation).

Note: This series is appropriate for regression equations of the type

$$Y=a+bX+cX^2,\ Y=ab^X \text{ and } Y=aX^b.$$

8. Using data from Table 16 of Appendix A giving the per cent of population making federal income tax returns and retail sales per capita, follow the instructions of problem J7 above, using the retail sales series as the dependent variable.

9. The data[*] given below are estimates of refined copper stocks in the United States and the wholesale prices of electrolytic copper as reported in Poor's *Industry and Investment Surveys:* "Copper Industry C4," pp. 19–20, November 1, 1939.

Jan. 1 of Year	Refined Copper Stocks (1000 short tons)	Price, Electrolytic Copper (monthly averages, producers' prices, delivered Connecticut points) (cents per pound)
1925	121.5	14.30
1926	62.0	14.05
1927	73.0	13.17
1928	85.5	14.81
1929	57.0	18.35
1930	153.0	13.23
1931	307.5	8.37
1932	462.3	5.79
1933	502.0	7.28
1934	406.5	8.66
1935	284.5	8.88
1936	175.0	9.71
1937	110.0	13.39
1938	179.0	10.22

(a) Construct a scatter diagram of the above data, using stocks as the independent variable.
(b) Calculate the least squares equation of regression showing the relationship $Y = a + bX + cX^2$, and plot on the scatter diagram.
(c) Calculate and plot the standard error of estimate of Y.
(d) Calculate and interpret the indexes of correlation and determination.

10. Using the data of problem J9 but correcting both series for trend and using deflated copper prices, follow the instructions in the above problem using as the function of the regression equation either $Y = ab^X$ or $Y = aX^b$.

11. The following data, taken from the *Census of Wholesale Distribution* for the United States for the year 1935, are for total net sales and operating expenses of

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[*] (Note to instructor: To avoid the element of spuriousness due to the presence of trend the instructor may, at his option, substitute the adjusted data as follows—

Copper stocks (1000 short tons): −41.9, −109.0, −105.6, −100.7, −136.8, −48.4, +98.5, +245.7, +277.8, +174.7, +45.1, −72.0, −144.6, −83.2

Copper prices (cents per pound): −.03, +.17, −.27, +1.81, +5.80, +1.12, −3.30, −5.43, −3.50, −1.67, −1.01, +.26, +4.39, +1.66)

wholesale distributors, classified by geographical regions and by states. All data are in millions of dollars.

	Net Sales	Operating Expenses
New England		
Connecticut	306	39
Maine	128	17
Massachusetts	1,806	174
New Hampshire	54	7
Rhode Island	181	21
Vermont	54	7
Middle Atlantic		
New Jersey	709	90
New York	10,847	1,006
Pennsylvania	2,741	269
East North Central		
Illinois	3,938	356
Indiana	655	64
Michigan	1,411	132
Ohio	2,029	211
Wisconsin	677	73
West North Central		
Iowa	628	51
Kansas	398	34
Minnesota	1,188	103
Missouri	1,812	179
Nebraska	528	38
North Dakota	134	14
South Dakota	99	12
South Atlantic		
Delaware	106	10
Florida	399	47
Georgia	605	58
South Atlantic (Cont.)		
Maryland	496	52
North Carolina	555	44
South Carolina	267	18
Virginia	303	50
West Virginia	231	27
East South Central		
Alabama	305	30
Kentucky	380	35
Mississippi	180	14
Tennessee	746	57
West South Central		
Arkansas	180	17
Louisiana	580	55
Oklahoma	404	36
Texas	1,532	143
Mountain		
Arizona	78	9
Colorado	324	31
Idaho	82	9
Montana	122	12
Nevada	15	2
New Mexico	46	5
Utah	154	17
Wyoming	26	3
Pacific		
California	2,975	355
Oregon	343	37
Washington	604	69

Source: U. S. Bureau of the Census: *Wholesale Distribution, 1935,* Vol. I, p. 55

(a) Rank the 48 states according to volume of net sales and according to size of operating expenses. Indicate the degree of agreement in the rankings by calculating the coefficient of rank correlation.

(b) Under what circumstances is this measure of association a satisfactory substitute for the coefficient of correlation?

12. Problem Q6 contains a table listing the population and the number of retail stores in 18 United States cities. Rank the 18 observations and indicate the degree of agreement in the rankings by calculating the rank correlation coefficient.

K. SIMPLE CORRELATION; GROUPED DATA

1. The correlation table following shows the relationship between total assets and total taxes of 100 American corporations with assets under $70,000,000 for the fiscal year 1939.

Total taxes (in millions of dollars)															
	3–3.99						1			1	2	1		1	
	2–2.99			1	1		1	2	2		3	1	3	2	1
	1–1.99			2	4	5	2	6	1	4	1	4	1		
	0–.99	2	16	10	7	3	2	3	1	1	1		1		
		0–4.9	5–9.9	10–14.9	15–19.9	20–24.9	25–29.9	30–34.9	35–39.9	40–44.9	45–49.9	50–54.9	55–59.9	60–64.9	65–69.9

Total assets (in millions of dollars)

Source: "Survey of Taxes for 1939"—American Federation of Investors, Inc.

(a) For both the X and Y series set up the columns necessary for the calculation of the summations needed to determine the linear equation of regression, the standard error of estimate of Y and the correlation coefficient by the least squares method.

(b) Restate the regression equation in terms of original units and to a zero origin. Restate the standard estimate of Y to original units.

2. Using the data from the correlation table in problem K1 above, determine by the product moment method the coefficient of correlation, the linear equation of regression and the standard error of estimate of Y. Restate the equation of regression in terms of original units and to a zero origin and restate the standard error of estimate in terms of original units.

3. The following correlation table relating federal income tax returns and retail sales is to be used for the calculation of a linear least squares regression equation, the standard error of estimate of Y and the coefficient of correlation,

(a) by the least squares method.

(b) by the product moment method.

Correlation Table Showing the Relation Between Per Cent of Population Making Federal Income Tax Returns and Estimated Retail Sales Per Capita by States for 1935*

X = Estimated retail sales per capita

Y = Per cent of population making federal income tax returns

Classes	$80–119	$120–159	$160–199	$200–239	$240–279	$280–319	$320–359	$360–399	$400–439	Totals
6–6.99%							1	1	1	3
5–5.99%					1	1	2			4
4–4.99%					1	2	1			4
3–3.99%					3	4	3			10
2–2.99%			2	5	6	2				15
1–1.99%		2	4	2						8
0– .99%	2	2								4
Totals	2	4	6	7	11	9	7	1	1	48

* Income tax returns made in 1936 on 1935 incomes.

Source: Compiled from data in Trull, Edna, *Resources and Debts of the Forty-Eight States, 1937,* page 98.

4. Using the correlation table below, follow the instructions of problem K1 above.

Correlation Table Showing the Relation Between Per Cent of Population Making Federal Income Tax Returns and Estimated Taxable Property Valuation Per Capita by States for 1936

X = Estimated taxable property valuation per capita

Y = Per cent of population making federal income tax returns

Classes	$400–599	$600–799	$800–999	$1,000–1,199	$1,200–1,399	$1,400–1,599	$1,600–1,799	$1,800–1,999	$2,000–2,199	$2,200–2,399	$2,400–2,599	Totals
6–6.99%										2	1	3
5–5.99%						1	1	2				4
4–4.99%						2	1	1				4
3–3.99%				1	2	5	1	1				10
2–2.99%			5	2	4	4						15
1–1.99%		4	2		1	1						8
0– .99%	1	3										4
Totals	1	7	7	3	7	13	3	4		2	1	48

Source: Compiled from data in Trull, Edna, *Resources and Debts of the Forty-Eight States, 1937,* page 98.

5. Using the correlation table given in problem K4, follow the instructions of problem K2 above.

6. Using the correlation table given in problem K3, calculate the regression equation $Y=a+bX+cX^2$, the standard error of estimate of Y and the index of correlation.

7. Using the correlation table given in problem K4 above, calculate the regression equation $Y=a+bX+cX^2$, the standard error of estimate of Y and the index of correlation.

8. Using the data of the correlation table given in problem K4 above,
   (a) calculate the correlation ratio with corrections for the number of column means;
   (b) calculate Zeta (using the uncorrected correlation ratio) and test with the standard error of Zeta.

9. Using the data of the correlation table given in problem K3, follow the instructions of problem K8.

L. SIMPLE CORRELATION; TIME SERIES

1. The data following are for steel ingot production and automobile production in the United States in monthly averages for the years 1922–1940.

Year	Steel Ingot Production (million short tons)	Automobile Production (thousand cars)
1922	3.24	212.0
1923	4.08	336.2
1924	3.45	300.2
1925	4.14	355.5
1926	4.41	358.4
1927	4.11	283.4
1928	4.72	363.2
1929	5.15	446.5
1930	3.72	279.7
1931	2.38	199.1
1932	1.26	114.2
1933	2.14	160.0
1934	2.43	229.4
1935	3.18	328.9
1936	4.46	371.2
1937	4.72	400.7
1938	2.65	207.4
1939	4.40	298.1
1940	5.58	372.4

Source: Survey of Current Business

(a) Plot each of the series arithmetically, determine the appropriate period for the calculation of trend and compute the least squares linear equation of trend for each series.

(b) Remove trend from each of the series (by division or by subtraction) and plot the percentages to trend (if trend is removed by division) or deviations from trend (if trend is removed by subtraction) on a scatter diagram.

(c) Determine the regression equation, the standard error of estimate and the correlation coefficient from the plotted data.

(d) Interpret specifically the results obtained in (c) bearing in mind the important limitations involved in a time series correlation.

2. The following data are the rates of earnings of the steel industry of the United States and ingot production expressed as percentages of capacity by years, 1918–1938.

Year	Net Income before Interest and Dividends as a Percentage of Capital Stock, Bonded Debt and Surplus	Ingot Production as a Percentage of Capacity
1918	9.0	84.6
1919	5.7	63.6
1920	7.2	75.7
1921	1.5	34.5
1922	3.0	60.9
1923	6.5	76.6
1924	5.0	63.8
1925	5.6	74.2
1926	6.8	83.5
1927	5.3	74.9
1928	6.4	83.9
1929	9.4	88.5
1930	4.5	62.5
1931	0.3	37.6
1932	−2.9	19.5
1933	−0.5	33.1
1934	0.5	37.4
1935	2.3	48.7
1936	4.6	68.4
1937	6.3	72.5
1938	0.5	39.6

Source: "Investigation of Concentration of Economic Power," Hearings before the Temporary National Economic Committee on the Iron and Steel Industry, part 18, pp. 10,423.

(a) same instructions as for problem L1.

3. The following data are the lapse rates of life insurance, the number of commercial failures and the Federal Reserve Board Index of Industrial Production, in monthly averages, for the years 1925–1940.

Year	Life Insurance Lapse Rate (1925–26 = 100)	Number of Commercial Failures (thousands)	Federal Reserve Board Index of Industrial Production (1935–39 = 100)
1925	100	1.77	104
1926	100	1.81	108
1927	103	1.93	106
1928	96	1.99	111
1929	97	1.91	119
1930	114	2.20	96
1931	119	2.36	81

1932	141	2.65	64
1933	139	1.67	76
1934	126	1.01	79
1935	122	1.02	90
1936	98	.80	105
1937	96	.79	110
1938	104	1.07	86
1939	92	.95	105
1940	91	1.13	122

Source: Lapse rate data from Life Insurance Sales Research Bureau; commercial failures and production index from *Survey of Current Business.*

(a) Same instructions as for problem L1, using any two of the above series.

4. Using the data given in problem L1,
   (a) determine the year-to-year differences for each of the two series.
   (b) plot the differences on a scatter diagram, using the differences in steel production as the independent series.
   (c) calculate and plot the linear least squares regression equation.
   (d) calculate and plot the standard error of estimate of Y.
   (e) determine the value of the correlation coefficient.
   If the estimate of steel production is 5.70 million short tons for 1941, what is your estimate of the production of automobiles for the same year? State the reliability of your estimate.

5. Using the data of L2,
   (a) determine the year-to-year differences for each of the two series.
   (b) plot the differences on a scatter diagram, using the differences in ingot production percentages as the independent variable.
   (c) calculate and plot the linear least squares regression equation.
   (d) calculate and plot the standard error of estimate of Y.
   (e) determine the value of the correlation coefficient.
   Estimate the net income percentage, given steel ingot production for 1939 as 65 per cent of capacity.

6. Using the data on commercial failures and industrial production of problem L3,
   (a) determine the year-to-year differences for each of the two series.
   (b) plot the differences on a scatter diagram, using the differences in the industrial production index series as the independent variable.
   (c) calculate the correlation coefficient by the product moment method.
   (d) determine the equation for the regression line and plot on the scatter diagram.
   (e) calculate and plot the standard error of estimate of Y.
   If the industrial production index for 1941 is estimated to be 130, what is the estimate of the number of commercial failures?

M. MULTIPLE CORRELATION

1. Table 16 of Appendix A gives data for the forty-eight states of the United States used in a study of the debt-paying capacity of the various states. It is desired to measure the association among the various series by means of a multiple correlation analysis. On the assumption that motor vehicle registration (X_1) is for the large part determined by the per cent of population making income tax returns (X_2), the value of taxable property (X_3) and the volume of savings deposits (X_4) calculate and interpret the following correlation measures:

(a) coefficient of multiple linear correlation ($R_{1.234}$).

(b) the equation of multiple regression

$$X_1 = a + b_{12.34}X_2 + b_{13.24}X_3 + b_{14.23}X_4.$$

(c) the standard error of estimate ($S_{1.234}$).

(d) the corrected coefficient of multiple correlation (corrected for the number of constants in the regression equation).

(e) an estimate of the sampling error of the multiple correlation coefficient (i.e., the standard error of $R_{1.234}$).

Make an estimate of the motor vehicle registration per 1000 population for a state where 3.5% of the population made federal income tax returns, where the estimated taxable property valuation per capita was $1,550 and where the savings deposits per capita was $150. Quote the reliability of your estimate.

2. Reduce the coefficients of net regression in the equation of multiple regression determined in M1(b) to comparable terms by expressing all variables in units of their respective standard deviations. Interpret precisely each of these beta coefficients.

3. Using the data of Table 16 of Appendix A, measure the association between per capita retail sales (X_1) and per cent of population making income tax returns (X_2), valuation of taxable property (X_3) and per capita savings deposits (X_4), by computing and interpreting the following multiple correlation measures:

(a) coefficient of multiple linear correlation ($R_{1.234}$).

(b) the equation of multiple regression

$$X_1 = a + b_{12.34}X_2 + b_{13.24}X_3 + b_{14.23}X_4.$$

(c) the standard error of estimate ($S_{1.234}$).

(d) the corrected coefficient of multiple correlation (corrected for the number of constants in the regression equation).

(e) an estimate of the sampling error of the multiple correlation coefficient (i.e., the standard error of $R_{1.234}$).

Make an estimate of the per capita retail sales (X_1) for a state where the per cent of population making federal income tax returns is 3.75, where the estimated taxable property valuation per capita is $1,500 and where the per capita savings deposits are $140. Quote the reliability of your estimate.

4. Data are given in Table 19 of Appendix A showing the average monthly prices of raw silk in the United States for the years 1923–1940. With these data are given six other economic series that are assumed to account for much of the variation in the price of raw silk. The six series are: U. S. imports of raw silk (X_2), world

visible supply of raw silk (X_4), raw silk supply in U. S. warehouses (X_5), index of textile manufactures in the United States (X_6), index of manufactures in the United States (X_7), raw silk deliveries to U. S. mills (X_8) and the wholesale price of rayon (X_9). Series X_2 represents the original raw silk price averages (X_1) deflated by the United States Bureau of Labor Statistics index of wholesale commodity prices. From these data, multiple correlation exercises are to set up involving combinations of the six independent variables as designated by the instructor. Suggested combinations for three-variable multiple correlation studies would involve the correlation of raw silk prices (X_1) with world visible supply (X_4) and the index of textile manufactures (X_6) or with United States raw silk supply (X_5) and the index of manufactures (X_7) or with deliveries to United States mills (X_8) and rayon prices (X_9).

(a) Using any two of the above independent series, selected by the instructor, compute the linear equation of multiple regression, the standard error of estimate and the coefficient of multiple correlation. Note that an element of spuriousness will materially influence the correlation measures unless each of the variables is first corrected for the influence of secular trend.

(b) Correct the standard error of estimate and the coefficient of multiple correlation for the number of observations and the number of constants in the regression equation.

5. Using three of the independent variables given in Table 19 of Appendix A, as selected by the instructor, follow the instructions under problem M4 for a four-variable multiple correlation analysis.

6. Perform a four-variable multiple correlation analysis from data in Table 19 of Appendix A, using deflated raw silk prices as the dependent variable and three independent variables assigned by the instructor. Analysis should follow the outline given in M1 omitting part (e). Trend corrections should be made before making the analysis. Distinguish between the results obtained from this analysis of deflated raw-silk prices and the measures calculated in problem M5 based on original silk prices.

7. Table 20 of Appendix A gives the wholesale prices of creamery butter in New York City, factory production, cold storage stocks and apparent consumption for the years 1919–1940. Using wholesale butter prices as the dependent variable, calculate the least squares linear multiple regression equation, the standard error of estimate (corrected for the number of observations and the number of constants in the regression equation) and the multiple correlation coefficient (corrected for the number of observations and the number of constants in the regression equation). Trend corrections should be made to the four variables before making the analysis.

8. Repeat the study indicated in problem M7 using deflated butter prices as the dependent series. Interpret specifically the calculated measures.

9. Data on market prices, earnings and net sales per share of stock of the Sears, Roebuck Company for the years 1926–1938 are reported in *Moody's Stock Survey*, August 21, 1939 as follows:

Year	Average Market Price of Stock	Earnings per Share	Net Sales per Share
1926	\$ $51\frac{3}{8}$	\$5.21	\$59.20
1927	71	5.96	64.00
1928	$139\frac{7}{8}$	6.28	74.40
1929	$130\frac{1}{2}$	6.62	89.70
1930	$71\frac{7}{8}$	3.01	74.60
1931	$46\frac{3}{4}$	2.47	64.60
1932	$23\frac{5}{8}$	0.53(d)	56.50
1933	$29\frac{3}{4}$	2.35	56.10
1934	$41\frac{1}{8}$	3.13	66.30
1935	$50\frac{3}{8}$	4.45	81.70
1936	$80\frac{1}{2}$	5.60	90.00
1937	$74\frac{1}{8}$	5.58	97.70
1938	$63\frac{1}{2}$	4.18	89.60

(d) deficit.

(a) After making the necessary corrections for trend in the three variables, conduct a three-variable multiple correlation study using the average market price of the stock as the dependent variable. The study should include the calculation and interpretation of the linear multiple regression equation, the standard error of estimate and the multiple correlation coefficient. The last two measures should be corrected for the number of observations and the number of constants in the regression equation. It is to be noted that although the number of observations is relatively small, this analysis serves to illustrate the multiple correlation technique with a minimum of routine calculation.

10. *Moody's Stock Survey*, December 4, 1939, reports the following data on four cigarette manufacturing companies for the years 1925–1938.

Year	Earnings of Four Cigarette Manufacturing Companies (million dollars)	Cigarette Tax Withdrawals (billions)	Cigarette Price (\$ per 1000)	Tobacco Costs (cents per lb.)
1925	68.4	80.0	6.40	20.2
1926	70.3	87.4	6.40	21.0
1927	73.6	97.2	6.40	21.7
1928	76.4	105.9	6.00	22.1
1929	85.7	119.0	6.40	22.2
1930	105.2	119.6	6.40	18.9
1931	110.6	113.4	6.85	15.5
1932	104.6	103.6	6.85	14.3
1933	57.7	111.8	5.50	14.0
1934	64.5	125.6	6.10	17.2
1935	68.1	134.6	6.10	18.0
1936	77.2	153.2	6.10	21.1
1937	77.5	162.6	6.25	21.3
1938	73.7	163.7	6.25	21.3

On the assumption that cigarette consumption (estimated by data showing cigarette tax withdrawals), the wholesale selling price of cigarettes and the cost of materials are important factors in the determination of earnings for the industry, a four-variable correlation analysis is to be performed. After correcting each of the variables for the effect of secular trend,

(a) calculate the linear multiple regression equation.
(b) determine the reliability of estimates by calculating the adjusted standard error of estimate.
(c) calculate and interpret the adjusted coefficients of multiple correlation and determination.

In 1939 cigarette tax withdrawals totaled 172.5 billion, the wholesale price of cigarettes averaged $5.51 per thousand and the average price of raw tobacco was 21.1 cents per pound. Make an estimate of the corresponding earnings in 1939 from the regression equation.

11. Calculate and interpret the coefficients of net or partial correlation from the data indicated in the study outlined in problem M1.

12. Calculate and interpret the three coefficients of net or partial correlation from the data indicated in the study outlined in problem M3 (M5 or M7).

13. From Table 19 of Appendix A giving data assumed to influence the wholesale price of raw silk, calculate coefficients of net or partial correlation from the following combination of variables.

(a) dependent variable: wholesale price of raw silk, undeflated; independent variables: world visible supply of raw silk and index of textile manufactures. Setting up an arithmetic sequence of whole numbers in consecutive order (1, 2, 3, 4, etc.) to represent time as one of the independent variables and using world visible supply of raw silk as a second independent variable, calculate the net correlation between wholesale prices of raw silk and indexes of textile manufactures.
(b) dependent variable: wholesale price of raw silk, deflated; independent variables: deliveries to U. S. mills, raw-silk supply in U. S. warehouses and a series to represent time (see (a) above). Holding the last two variables named constant, calculate and interpret precisely the coefficient of partial correlation between deflated raw-silk prices and deliveries to U. S. mills.

14. Reduce the coefficients of net regression in the multiple regression equation determined in M3 (M4, M5, M6, M7, M8, M9 or M10) to comparable terms by expressing all variables in units of their respective standard deviations. Interpret precisely each of these beta coefficients.

N. THE NORMAL CURVE OF ERROR

DESCRIPTION OF FREQUENCY DISTRIBUTIONS BY THE MOMENT METHOD

1. The following distribution shows the weekly earnings of skilled explosives workers in the United States in October, 1937.

Weekly Earnings	Number of Workers
$ 5– 9.99	12
10–14.99	10
15–19.99	33
20–24.99	117
25–29.99	319
30–34.99	563
35–39.99	464
40–44.99	319
45–49.99	125
50–54.99	58
55–59.99	17
60–64.99	10
65–69.99	6
70–74.99	2
Total	2,055

Source: Monthly Labor Review, Aug. 1938, p. 392

(a) Describe the distribution by means of the following attributes:
  (1) Central tendency: mean and mode (subtracting the modal divergence from the mean)
  (2) Dispersion: standard deviation (without Sheppard's correction)
  (3) Skewness: Chi
  (4) Kurtosis: B_2

(b) Using the calculated values of B_1 and B_2, determine the curve type according to Pearson's classification.

2. The following distribution gives the percentage of calls involving health service made by 132 official nursing agencies in the United States during the year 1939.

Per Cent of Calls Involving Health Service	Number of Agencies
0– 9.9	2
10–19.9	5
20–29.9	17
30–39.9	20
40–49.9	28
50–59.9	21

60–69.9	21
70–79.9	9
80–89.9	6
90–99.9	3

Source: National Organization of Public Health Nurses: *Yearly Review, 1939*

(a) Describe the distribution according to instructions in problem N1(a).
(b) Determine the curve type according to Pearson's classification.

3. The following distribution gives the percentage of calls involving non-communicable diseases made by 185 non-official visiting nurse agencies in the United States during 1939.

Per Cent of Calls Involving Non-Communicable Diseases	Number of Agencies
0– 9.9	4
10–19.9	9
20–29.9	20
30–39.9	41
40–49.9	45
50–59.9	34
60–69.9	19
70–79.9	11
80–89.9	1
90–99.9	1
	185

Source: National Organization of Public Health Nurses: *Yearly Review, 1939*

(a) Describe the distribution according to instructions in problem N1(a).
(b) Determine the curve type according to Pearson's classification.

4. The following tabulation gives the average weekly earnings during the month of December, 1934 of male employees in three occupational classes of the United States bread-baking industry (in the North).

Average Weekly Earnings	Cake-Makers (number)	Mixers (number)	Miscellaneous Skilled (number)
0–$3.99	3	—	—
4– 7.99	10	1	1
8–11.99	10	2	2
12–15.99	16	6	5
16–19.99	48	25	34
20–23.99	81	59	44
24–27.99	135	123	32
28–31.99	90	78	23

32–35.99	45	70	12
36–39.99	29	36	3
40–43.99	16	14	1
44–47.99	7	8	1
48–51.99	4	7	—
	494	429	158

Source: Wages, Hours, and Working Conditions in the Bread-Baking Industry, 1934, p. 138, U. S. Dept. of Labor, Bureau of Labor Statistics

(a) Describe the distribution of earnings of cake-makers according to instructions in problem N1(a) and determine the Pearsonian curve type.

(b) Describe the distribution of earnings of mixers according to instructions in problem N1(a) and determine the Pearsonian curve type.

(c) Describe the distribution of earnings of miscellaneous skilled workers according to instructions in problem N1(a) and determine the Pearsonian curve type.

5. The following tabulation gives the distribution of heights of 3,599 First World War draft recruits according to chest measurement.

Height in Inches	Number of Recruits having Chest Measurements of		
	37 inches	38 inches	39 inches
59	6	5	2
60	1	2	—
61	8	1	3
62	17	10	3
63	27	17	14
64	64	22	26
65	105	32	51
66	157	74	64
67	246	100	98
68	289	135	134
69	266	134	148
70	267	109	109
71	192	77	106
72	102	65	63
73	66	31	40
74	25	14	15
75	20	5	4
76	6	5	3
77	1	3	3
78	1	1	—
79	4	—	1
	1,870	842	887

Source: The Medical Department of the United States Army in the World War, Volume XV, *Statistics*, Part I, p. 442

(a) Describe the distribution of heights of recruits with 37-inch chest measurements by means of the following attributes:
  (1) Central tendency: mean and mode (subtracting the modal divergence from the mean)
  (2) Dispersion: standard deviation (with Sheppard's correction)
  (3) Skewness: Chi
  (4) Kurtosis: B_2

(b) Using the calculated values of B_1 and B_2, determine the curve type according to Pearson's classification.

6. Describe the distribution of heights of recruits with 38-inch chest measurements (from the data in problem N5) according to instructions in problem N5 and determine the Pearsonian curve type.

7. Describe the distribution of heights of recruits with 39-inch chest measurements (from the data in problem N5) according to instructions in problem N5 and determine the Pearsonian curve type.

FITTING A NORMAL CURVE

8. The following tabulation gives the distribution of heights of United States Army recruits, by States, at demobilization after World War I.

Comparative Frequency Distribution of Height, by States, of White and Colored Troops at Demobilization

Height in Centimeters	Alabama	Arkansas	Georgia	Kansas	Maryland	Michigan	New York	Pennsylvania	Washington
148–151.9	2	2	2	—	—	2	12	8	—
152–155.9	1	6	6	2	8	31	91	96	3
156–159.9	13	19	29	2	46	95	383	374	20
160–163.9	87	102	151	42	95	324	1,020	1,147	98
164–167.9	171	291	355	117	204	654	1,798	2,136	219
168–171.9	361	530	662	198	268	885	2,193	2,622	395
172–175.9	497	618	807	275	266	828	1,897	2,277	485
176–179.9	429	539	718	191	157	570	1,110	1,387	467
180–183.9	238	307	408	112	61	220	477	576	202
184–187.9	86	116	203	54	23	81	169	175	99
188–191.9	35	36	45	14	9	21	47	59	27
192–195.9	8	7	8	4	—	4	7	13	8
196–199.9	2	3	2	1	1	—	1	3	2
200–203.9	—	—	—	—	—	—	1	—	—
204–207.9	—	—	1	—	—	—	1	1	—
Totals	1,930	2,576	3,397	1,012	1,138	3,715	9,207	10,874	2,025

Source: The Medical Department of the United States Army in the World War, Volume XV, *Statistics*, Part I, p. 566

(a) Compute the theoretical frequencies of a normal curve for recruits from a state to be designated by the instructor.

(b) Plot a frequency polygon of the theoretical distribution on a chart showing a polygon of the actual series.

9. The following percentage distribution gives the average hourly earnings of

female workers in the cotton textile industry in the United States during the month of August, 1938.

Average Hourly Earnings	Per Cent of Female Workers
$.075–.124	.6
.125–.174	1.6
.175–.224	4.0
.225–.274	8.0
.275–.324	24.9
.325–.374	31.9
.375–.424	16.2
.425–.474	7.7
.475–.524	3.6
.525–.574	1.1
.575–.624	.3
.625–.674	.1
	100.0

Source: Monthly Labor Review, December, 1938, p. 1244

(a) Compute the theoretical percentages of a normal curve based on the above distribution.

(b) Plot a frequency polygon of the theoretical percentages on a chart showing a polygon of the original series.

10. The following table, constructed from a chart published in Sears, Roebuck & Company's mail order catalog, permits the transformation of foot length in centimeters to corresponding shoe sizes for men. On the assumption that customers of this company have a mean foot length of 24.9 centimeters and a standard deviation of .645 centimeters, estimate the number of pairs of shoes of each size to be ordered in a 500,000 pair order from a manufacturer.

Shoe Size Number	Shoe Size in Centimeters
6	22.15–22.54
6½	22.55–22.99
7	23.00–23.39
7½	23.40–23.84
8	23.85–24.29
8½	24.30–24.74
9	24.75–25.14
9½	25.15–25.64
10	25.65–26.04
10½	26.05–26.54
11	26.55–26.99
11½	27.00–27.49
12	27.50–27.94

11. The following table gives the blouse size corresponding to waist measurements used by the United States Army in the first World War.

Blouse Size	Waist Circumference in Centimeters
1	60– 63
2	64– 67
3	68– 71
4	72– 75
5	76– 79
6	80– 83
7	84– 87
8	88– 91
9	92– 95
10	96– 99
11	100–103
12	104 and over

On the assumption that the average measurements of recruits at the present time conform closely to the army measurements during the first World War, construct an order for 50,000 blouses to be distributed to men recruited from

(a) California—mean waist measurement 78.19 centimeters, standard deviation 6.344 centimeters.

(b) Connecticut—mean waist measurement 75.59 centimeters, standard deviation 5.420 centimeters.

(c) Maine—mean waist measurement 77.52 centimeters, standard deviation 5.883 centimeters.

O. THE CHI-SQUARE TEST

GOODNESS OF FIT

1. The following tabulation gives the distribution of male, unskilled workers by annual earnings in the meat-packing industry of the Southern Wage District in 1937 classified according to the length of employment.

Annual Earnings	Number of employees whose work extended over		
	6 months or more	9 months or more	12 months or more
Under $200	1	1	—
$200 and under $400	41	9	4
$400 and under $600	92	32	11
$600 and under $800	153	105	42
$800 and under $1,000	270	253	136
$1,000 and under $1,200	231	231	157
$1,200 and under $1,400	86	86	73
$1,400 and under $1,600	14	14	12
$1,600 and under $1,800	2	2	2
$1,800 and under $2,000	1	1	1
$2,000 and under $2,200	—	—	—
$2,200 and under $2,400	1	1	1
Total number of workers	892	735	439

Source: "Annual Earnings in Meat Packing, 1937," *Monthly Labor Review*, December, 1939

(a) From one of the distributions above (to be assigned by the instructor) calculate the mean and standard deviation and determine the theoretical frequencies for a normal curve.

(b) Employing the Chi-square test, determine the probability that the discrepancies between the observed and theoretical frequencies could have arisen as a result of chance.

2. The following tabulation gives the number of unskilled workers in the wood household-furniture branch of the furniture manufacturing industry of the United States according to weekly earnings in October, 1937.

Average Weekly Earnings	Number of Workers Employed as a Whole	Number of Workers Employed in the North
Under $5	209	83
$5 and under $10	838	291
$10 and under $15	2,005	910
$15 and under $20	1,857	1,433
$20 and under $25	859	810
$25 and under $30	230	230
$30 and under $35	47	46

$35 and under $40	9	9
$40 and under $45	4	4
$45 and under $50	3	2
Total	6,061	3,818

Source: "The Wage and Hour Structure of the Furniture-Manufacturing Industry, October, 1937," U. S. Dept. of Labor, Bureau of Labor Statistics

(a) From each of the distributions above calculate the mean and standard deviation and determine the theoretical frequencies of the corresponding normal curves.

(b) Employing the Chi-square test, determine the probability that the discrepancies between the observed and theoretical frequencies could have arisen as a result of chance.

INDEPENDENCE OF PRINCIPLES OF CLASSIFICATION

3. *Number of Industrial Injuries in the United States During 1937 According to Industrial Groups and Extent of Injury.*

Industrial Group	Total	Permanent	Temporary
Agriculture	266,000	13,500	252,500
Mining and Quarrying	105,900	3,300	102,600
Construction	388,500	20,600	367,400
Manufacturing	361,900	27,900	334,000
Public Utilities	16,200	700	15,500
Trade—wholesale and retail	238,400	32,000	206,400
Railroads	39,800	1,400	37,900
Miscellaneous Transportation	41,600	2,200	39,400
Services and Miscellaneous Industries	361,100	25,100	336,000
Total	1,818,400	126,700	1,691,700

Source: "Industrial Injuries in the United States During 1937," *Monthly Labor Review,* March, 1939, p. 4

From the above records it is desired to determine whether the classification according to industrial groups and the classification according to the length of industrial injury are independent or related. On the assumption that the permanence or temporariness of injury is the result more of chance than of industrial occupation set up theoretical frequencies in accordance with that hypothesis and apply the Chi-square test (with reference to Table V of Appendix B).

4. A study of the sex ratio resulting from 8,251,013 live births in the native white population of the United States from 1931 through 1935 has led to the conclusion by the Metropolitan Life Insurance Company of New York that "the ratio of males to females at birth diminishes as the age of mothers increases." The data given in support of the observation follow.

Number of Male and Female Live Births, According to Age of Mother, in the Native White Population of the United States, 1931–35

Age of Mother	Male Live Births	Female Live Births	Total	Males per 1000 Females
10–14 yrs.	2,911	2,686	5,597	1,084
15–19	501,559	473,171	974,730	1,060
20–24	1,332,521	1,255,918	2,588,439	1,061
25–29	1,127,870	1,066,589	2,194,459	1,057
30–34	710,228	672,490	1,382,718	1,056
35–39	414,696	393,905	808,601	1,053
40–44	139,446	133,165	272,611	1,047
45 and over	12,207	11,651	23,858	1,048
Total	4,241,438	4,009,575	8,251,013	

Source: Metropolitan Life Insurance Company *Statistical Bulletin,* Vol. 20, No. 4, April, 1939

Determine, by means of the Chi-square test, whether the classification according to sex is related to or independent of the age of the mothers.

5. A tabulation of results of all strikes ending in January and February, 1941, in relation to the major issues involved resulted in the following double classification.

Major Issue Involved	Number of Strikes Resulting in		
	Substantial Gains to Workers	Little or No Gain to Workers	Total
Wages and hours	49	19	68
Union organization	100	42	142
Total	149	61	210

Source: Monthly Labor Review, June, 1941, p. 1477

It is desired to determine whether the issue involved in strikes is related to the results achieved by the strikes. On the assumption that the two principles of classification are independent, determine a set of observations in accordance with the hypothesis and apply the Chi-square test.

6. A tabulation of the duration of strikes ending in the month of March, 1940, in relation to the type of industry involved, resulted in the following double classification.

Type of Industry	Number of Strikes with Duration of Less Than One Month	More Than One Month	Total
Manufacturing	85	13	98
Non-manufacturing	51	9	60
Total	136	22	158

Source: Monthly Labor Review, July 1940, p. 152

On the assumption that the type of industry is independent of the duration of strikes set up theoretical frequencies to accord with that hypothesis and apply Chi-square test.

TEST OF HOMOGENEITY

7. A national survey of the incidence of colds and influenza covering 3,091 adults in five geographical regions of the United States during the winter of 1940–1941 resulted in the following tabulations.

Geographical Region	Number of Persons Who Had Colds During the Winter	Had No Colds During the Winter	Total Number Interviewed
New England and Middle Atlantic	674	379	1,053
East Central	426	243	676
West Central	328	184	512
South	294	115	409
Rocky Mountains and Pacific	256	185	441
Total	1,978	1,113	3,091

Geographical Region	Number of Persons Who Had Influenza or Grippe* During the Winter	Did Not Have Influenza or Grippe During the Winter	Total Number Interviewed
New England and Middle Atlantic	242	811	1,053
East Central	149	527	676
West Central	133	379	512
South	119	290	409
Rocky Mountains and Pacific	115	326	441
Total	758	2,333	3,091

Source: Survey conducted by the American Institute of Public Opinion

* Defined as a heavy cold or fever.

A study is to be made to determine whether, in each of the above tabulations, the population interviewed represents a homogeneous whole or whether the division into those persons having and those persons not having the particular ailment results in significant differences from region to region. Set up theoretical frequencies on the assumption of homogeneity and determine whether the discrepancies between observed and theoretical frequencies are to be attributed to chance.

8. In a "Study of Markets and Magazines" by *Modern Magazines* in 1937, the following tabulation is reported of the purchase of new and used automobiles in 13 American cities.

City	Bought New Cars	Bought Used Cars
Buffalo	1,772	1,235
Akron	1,445	991
Cleveland	2,510	1,618
Columbus	1,188	770
Cincinnati	2,043	1,158
Toledo	1,281	891
Indianapolis	1,415	863
Evansville	629	496
Knoxville	818	465
Birmingham	1,252	701
Fort Worth	944	769
Houston	1,341	901
San Francisco	1,624	1,231
Total	18,262	12,089

By means of the Chi-square test determine whether the 13 cities represent a homogeneous whole or whether the division into new and used car purchases results in differences from city to city that are to be considered significant.

9. A survey to determine the make of automobile preferred by purchasers of new automobiles was made in fourteen United States cities. The following table gives the preference for Buicks out of the total car sales for three makes (including Buicks) according to the residence of the buyer.

City	Total Sales of 3 Makes of Cars	Buick Sales
Buffalo	564	218
Washington	688	273
Pittsburgh	800	285
Akron	399	88
Cleveland	699	236
Columbus	276	90
Cincinnati	513	152
Toledo	302	76

Indianapolis	374	125
Evansville	157	52
Knoxville	185	71
Birmingham	217	82
Fort Worth	182	69
Houston	241	71
Total	5,597	1,888

Source: "Study of Markets and Magazines," *Modern Magazines*, 1937

On the assumption that these fourteen cities constitute a homogeneous population as regards preference for Buick automobiles, set up theoretical frequencies for the two components needed in the computation of Chi-square (one derived from a comparison of the actual and theoretical frequencies of Buick sales and one derived from a comparison of the actual and theoretical frequencies of non-Buick sales). What is the probability that the differences between the theoretical and observed frequencies reflect the play of chance rather than conscious choice on the part of the buyers?

P. MEASURES OF RELIABILITY

1. *Distribution of Women Workers by Average Hourly Earnings and Class of Work in the Rubber Boot and Shoe Industry of the United States, May, 1940*

Average Hourly Earnings (cents per hour)	Class of Work	
	Production	Non-production
27.5 and under 32.5	260	15
32.5 and under 37.5	463	93
37.5 and under 42.5	898	304
42.5 and under 47.5	1,129	313
47.5 and under 52.5	1,480	212
52.5 and under 57.5	1,677	88
57.5 and under 62.5	786	63
62.5 and under 67.5	231	45
67.5 and under 72.5	56	18
72.5 and under 77.5	21	7
77.5 and under 82.5	7	11
82.5 and under 87.5	0	4
87.5 and under 92.5	7	1
Total	7,015	1,174

Source: Monthly Labor Review, June, 1941

(a) From the distribution showing the number of women workers in production calculate the following descriptive measures:
   (1) mean
   (2) median
   (3) quartile deviation
   (4) standard deviation

(b) Calculate the standard error of each of the measures computed in (a) and give precise interpretations of the values.

2. From the distribution in problem P1 showing the number of women workers in non-production work in the rubber boot and shoe industry compute the probable errors of the mean, the median, the quartile deviation and the standard deviation. Interpret each measure of reliability.

3. For each of the distributions given in problem P1 compute the standard errors of the mean, the median and the standard deviation. Compute the standard error of the difference between each pair of measures and quote the probability that such a difference may be attributed to errors of sampling.

4. The following tabulation shows the number of male workers in the wool carpet and rug industry of the United States, by skill, classified according to average hourly earnings in 1939.

Average Hourly Earnings (cents per hour)	Number of Workers	
	Skilled	Unskilled
22.5 and under 32.5	81	15
32.5 and under 42.5	377	587
42.5 and under 52.5	1,801	2,103
52.5 and under 62.5	2,621	1,957
62.5 and under 72.5	1,853	233
72.5 and under 82.5	413	131
82.5 and under 92.5	199	20
92.5 and under 102.5	37	10
Total	7,382	5,056

Source: Monthly Labor Review, July, 1940

(a) Compute the mean, quartile deviation and standard deviation for each of the above distributions and test the apparent differences between the means, the quartile deviations and standard deviations for significance.

(b) What is the probability that the mean hourly earnings for skilled workers as computed from the above sample is within one-fifth of a cent of the true mean earnings for all skilled workers in the industry? How large must the sample be to reduce the standard error of the mean by one-half?

5. Using the frequency distribution given in problem C13 (or C15 or C16) compute and give precise interpretations of each of the following:

(a) standard error of the mean
(b) probable error of the median
(c) standard error of the standard deviation
(d) standard error of the difference between the mean and the mode

6. An investigation involving brand preference for baking powder reported by *Modern Magazines* in a "Study of Markets and Magazines" and covering a survey of 1,383,468 persons indicated that 247,112 or 17.9% chose Royal baking powder compared to 210,803 persons or 15.2% who chose Davis OK powder. Calculate the standard error of the difference between the two percentages and determine if the apparent greater preference for Royal powder is to be considered significant.

7. A breakdown of the persons selecting Camel cigarettes as their preferred brand into income groups resulted in the following distribution:

Weekly Income Group	Number of Persons Interviewed	Number in Group Choosing Camels
Less than $20	41,329	14,520
$20–39	410,242	151,356
40–59	375,631	132,789
60–79	147,507	44,801
80 and over	191,424	60,139

Source: Modern Magazines, "Study of Markets and Magazines," 1937

(a) Calculate the standard error of the difference between two percentages for the first and second income groups and determine if the apparent difference in percentages of persons choosing Camel cigarettes is to be considered significant.

(b) Calculate the standard error of the per cent choosing Camels in the highest income group and determine whether this per cent differs significantly from the per cent for the total number interviewed.

8. A study of brand preference for dry cereals reported by *Modern Magazines* in a "Study of Markets and Magazines," 1937, reveals the following data:

Weekly Income Group	Number of Persons Interviewed	Number in Group Choosing Kellogg's Corn Flakes
Less than $20	44,894	10,827
20–39	435,077	122,456
40–59	396,308	127,682
60–79	156,563	46,668
80 and over	206,752	65,707

Determine whether the difference in the percentages of persons in the second and fourth income groups choosing Kellogg's Corn Flakes as the preferred dry cereal is to be considered significant.

9. The assignment in problem J5 requires a simple correlation study to be made of the relationship between the per cent of population making federal income tax returns and the retail sales per capita for the 48 states of the United States (data in Table 16 of Appendix A). Compute and interpret the standard errors of the correlation coefficient and the coefficient of regression.

10. The assignment in problem J6 requires a simple linear correlation study to be made of the relationship between the assets and taxes paid of 100 corporations (data of Table 17 of Appendix A). Compute and interpret the standard errors of the correlation coefficient and the coefficient of regression.

11. Compute and interpret the standard errors of the correlation coefficient and the coefficient of regression calculated in problem K1 (or K4).

12. Problem K6 requires the calculation of the index of correlation from data on the per cent of population making federal income tax returns and the estimated retail sales per capita for the 48 states of the United States. Compute and interpret specifically the standard error of the correlation index.

13. The index of correlation showing the degree of relationship between the per cent of population making federal income tax returns and the estimated taxable property valuation per capita for the 48 states of the United States is required in problem K7. Compute and interpret the standard error of the correlation index.

(Note to the instructor: The frequency distributions to be constructed in problems C1, C5 and C9 and the distributions given in problems C13, C15, C16, C17, C18, N1, N2, N3, O1 and O2 may be utilized for supplementary exercises involving the calculation of measures of reliability of the various descriptive measures of central tendency and dispersion. The distributions given in problems N4, N5 and N8 provide data for the calculation and interpretation of standard errors of the difference between means, medians, standard deviations, etc.)

Q. SMALL SAMPLES

1. The following data are the net earnings per share in 1939 of 19 leading stock issues as reported in the *Magazine of Wall Street,* April 20, 1940.

Name of Company	Net Earnings Per Share
Douglas Aircraft	$4.81
G. L. Martin	3.75
United Aircraft	3.53
Ex-Cell-O Corporation	2.21
Ingersoll-Rand	6.00
General Electric	1.48
Westinghouse Electric	5.18
Bethlehem Steel	5.75
United States Steel	1.84
Chrysler	8.48
General Motors	4.06
International Paper and Power	.14
Sutherland Paper	2.52
Baldwin Locomotive Works	.41
Pullman	1.05
Socony Vacuum	1.25
Standard Oil of New Jersey	3.00
Paramount Pictures	.63
Loew's	5.57

(a) Calculate the standard error of the mean from the above series.

(b) By reference to the *t*-table (Table II of Appendix B) determine whether the mean of this sample deviates significantly from an assumed mean of $3.50 for all companies.

2. The following data are the number of pages of advertising contained in a sample of 9 women's magazines for the month of April, 1940.

Name of Magazine	Number of Pages of Advertising
Harper's Bazaar	88
Ladies' Home Journal	75
Woman's Home Companion	73
McCall's	73
Parents' Magazine	66
Household Magazine	22
Photoplay	19
Junior League Magazine	13
Woman's World	8
Average	48.56

Source: Printer's Ink, p. 84, April 12, 1940

(a) Calculate the standard error of the mean of the above sample.

(b) By reference to the t-table (Table II of Appendix B) determine whether this sample mean is to be judged significantly greater than 30.89, the average number of pages of advertising for 70 magazines reported by the same source.

3. The following table gives the mean annual wage for employees in manufacturing industries in the north central and south central geographic divisions of the United States in 1917.

	Mean Annual Wage*
North Central division	
Ohio	$1,379.49
Indiana	1,283.32
Illinois	1,289.98
Michigan	1,493.68
Wisconsin	1,266.15
Minnesota	1,194.25
Iowa	1,122.50
Missouri	1,084.33
North Dakota	1,118.43
South Dakota	1,103.62
Nebraska	1,129.45
Kansas	1,187.09
Average	1,221.02
South Central Division	
Kentucky	960.16
Tennessee	808.81
Alabama	798.48
Mississippi	531.64
Arkansas	663.47
Louisiana	791.55
Oklahoma	1,163.75
Texas	1,023.20
Average	842.63

Source: Statistical Abstract of the United States, 1940, p. 840

* Derived by dividing the total wages paid by the average number of wage-earners employed during the year.

(a) Calculate the standard errors of the means of the above samples and by reference to the t-table (Table II of Appendix B) determine whether either of the sample means differs significantly from the mean annual wage for the entire United States manufacturing industry which was $3,170.49 for the same year.

4. The following table gives the average liability for industrial and commercial

failures in 1939 by selected industries in retail and wholesale trade in the United States.

Retail Trade Industries	Average Liability
Foods	$ 4,989
Farm supplies	7,143
General merchandise	10,020
Apparel	7,188
Furniture and house furnishings	9,100
Lumber, building materials, hardware	12,233
Automotive products	9,767
Restaurants	7,587
Drugs	8,069

Wholesale Trade Industries	Average Liability
Farm products, food, groceries	$16,281
Clothing and furnishings	10,937
Dry goods and textiles	14,864
Lumber, building materials, hardware	21,656
Chemicals and drugs	13,179
Fuels	22,621
Automotive products	11,817
Supply houses	10,851

Source: Statistical Abstract of the United States, 1940, p. 312

(a) For each of the two classifications of industry compute the standard error of the sample mean and, by reference to the t-table (Table II of Appendix B), determine whether either of the sample means differs significantly from the mean liability for the United States as a whole, $12,359.

5. At the annual regatta held by the Intercollegiate Rowing Association at Poughkeepsie, New York, on June 25, 1941, the heights of the varsity crew men of eight of the universities participating were as follows:

Univ. of California	6'2"	6'2"	6'4"	5'11"	6'3½"	6'4"	6'2¼"	6'5"
Univ. of Washington	6'8"	6'1"	6'3"	6'4"	6'4"	6'2"	6'2"	6'3"
Columbia University	6'1"	6'1"	6'2"	6'3"	6'4"	6'0"	6'2"	6'2"
Cornell University	6'3"	6'1"	6'2"	6'3"	6'4"	6'0"	6'2"	6'2"
Mass. Inst. of Technology	5'11¾"	6'0"	6'1"	6'1½"	6'1½"	6'2¾"	6'2"	6'1¾"
Princeton University	6'0"	6'0"	6'1"	5'11"	6'2"	6'4"	6'5"	6'3"
Syracuse University	6'2"	6'0"	6'2"	6'4"	6'4"	6'2"	6'3"	6'4"
Rutgers University	6'1"	6'1"	6'5"	6'1½"	6'4"	6'3"	6'1"	6'1"

(a) By means of the t-test, determine whether there is a significant difference between the mean height of crew men from western universities and the mean height of crew men from eastern universities.

6. The following table gives the metropolitan population and the number of retail stores in the '100% district' of 18 United States cities.

City	Number of Retail Stores in District	Population of Metropolitan Area (in thousands)
Detroit	99	2,105
Pittsburgh	76	1,954
Cleveland	63	1,195
Providence	37	964

Baltimore	168	949
Buffalo	82	821
Cincinnati	20	759
Milwaukee	79	743
Washington	110	621
Kansas City	72	608
Seattle	24	421
Birmingham	39	383
Portland	37	379
Atlanta	20	371
Dayton	66	252
Grand Rapids	56	207
Tulsa	79	183
Flint	65	180

Source: The Journal of Land Public Utility Economics, August, 1939

(a) By reference to the t-table (Table II of Appendix B), test whether the coefficient of correlation computed from the above sample differs significantly from zero.

7. *Earnings and Dividends per Share of Common Stock for Eight United States Aviation Manufacturing and Transport Companies in 1940*

Name of Company	Per Share Earnings	Dividend Disbursements
Consolidated Aircraft	$2.30	$2.00
Curtis-Wright Corp.	1.37	.50
Lockheed Aircraft Corp.	3.17	1.50
Martin (Glenn L.) Corp.	5.63	2.00
N. Amer. Aviation, Inc.	2.06	1.25
Sperry Corp.	3.85	2.00
United Aircraft Corp.	4.95	3.50
American Airlines, Inc.	4.12	1.00

Source: Fitch Revisions, Vol. 28, #119, Section Two. Fitch Investors' Service

(a) Calculate the correlation coefficient for the above sample and, by reference to the t-table (Table II of Appendix B); determine whether the coefficient from this limited sample differs significantly from zero.

(b) Check the conclusion arrived at in part (a) by reference to the table (Table III of Appendix B) prepared by R. A. Fisher showing the values of correlation coefficients at stated levels of significance. Using a standard of significance of $P = .01$, how high a coefficient would be required for a sample of eight observations before it could be accepted as significant?

8. The following data constitute samples of average hourly wage rates of union members in the printing trades in 46 United States cities grouped according to the population of the city.

Mean Hourly Wage Rates of Union Members in the Printing Trades

Group C (Cities with population of 250,000 to 500,000)				Group D (Cities with population of 100,000 to 250,000)				
$1.183	$1.063	$.987	$.894	$1.178	$1.040	$.992	$.935	$.926
1.169	1.040	.982	.893	1.137	1.020	.991	.935	.922
1.140	1.028	.974	.851	1.119	1.018	.962	.933	.874
1.103	1.026	.962	.844	1.098	1.017	.946	.930	.830
1.083	1.024	.914	.835	1.052	.995	.935	.926	.817
			.765					

Source: Monthly Labor Review, December, 1939, p. 1504

(a) Determine whether there is a significant difference in the mean hourly wage rates of the two groups of cities by applying the t-test.

9. The following data are the average annual earnings of employees in 41 northern furniture factories of the United States in 1936, subdivided according to the type of product manufactured.

Average Annual Earnings of Employees in Northern U. S. Furniture Factories

Mean Earnings in Factories Producing Case Goods				Mean Earnings in Factories Producing Upholstered Furniture			
$ 958	$1,406	$1,252	$1,300	$1,166	$1,319	$1,197	$ 943
1,235	1,002	831	982	955	1,293	1,242	1,188
1,070	1,520	822	1,255	1,548	1,277	1,068	1,223
1,080	1,016	955	1,153	1,374	1,118	1,379	1,689
1,027	1,095	1,059	1,293	1,094	980	1,181	1,183
							1,241

Source: Monthly Labor Review, April, 1939, p. 784

(a) By reference to the t-table (Table II of Appendix B), determine whether there is a significant difference between the mean annual earnings of the employees in the two kinds of factories.

10. The following table gives the earnings and dividends per share of common stock in 1940 for 10 companies manufacturing chemicals and for 10 companies manufacturing food products.

Type of Manufacturing

Chemicals		Food Products	
Earnings per Share	Dividends per Share	Earnings per Share	Dividends per Share
$ .91	$ .25	$ .77	$ .60
5.71	4.25	1.47	1.10
6.07	4.60	1.90	1.00

4.01	2.85	1.50	1.20
2.47	2.50	1.91	1.00
3.36	3.00	3.18	2.00
1.72	1.50	1.72	1.40
4.04	3.00	1.73	.80
1.36	.50	1.99	1.20
2.38	1.75	6.77	5.00

Source: Fitch Revisions, Vol. 28, pp. 89 and 107, Section Two. Fitch Investors' Service

(a) Compute the coefficient of correlation for each of the two samples.
(b) Transform the coefficients to corresponding z's (by reference to Table IV of Appendix B) and determine whether the difference in z's is to be considered significant.
(c) If it is concluded that the two samples were drawn from the same parent population, combine the two coefficients into a single weighted coefficient.

11. The following table gives the net sales and average number of employees in 1935 in selected kinds of business of the wholesale trade of the United States, according to the type of operation.

	Type of Operation			
Kind of Business	Manufacturers' Sales Offices (without Stocks)		Agents and Brokers	
	Net Sales (millions of dollars)	Employees, Average for Year (thousands)	Net Sales (millions of dollars)	Employees, Average for Year (thousands)
Automotive	53.5	.69	66.7	1.00
Beers, wines and liquors	81.3	.93	21.5	.23
Chemicals and paints	190.2	2.48	98.4	1.08
Drugs and drug sundries	9.9	.52	26.9	.64
Furniture and house furnishings	56.1	.60	75.3	1.18
Hardware	10.1	.22	32.5	.65
Jewelry and optical goods	6.5	.15	6.7	.12
Paper and its products	136.8	1.07	39.0	.54
Plumbing, heating equipment and supplies	23.0	.67	25.8	.89
Total	567.4	7.33	392.8	6.33

Source: Statistical Abstract of the United States, 1940

(a) Compute the coefficients of regression and correlation from each of the two samples.
(b) By means of the t-table (Table II of Appendix B) determine whether the difference in regression coefficients is to be considered significant.
(c) Transform the correlation coefficients to corresponding z-values (Table IV of Appendix X) and determine whether the apparent difference in degree of correlation for the two samples is to be considered due to factors other than chance.

Coupon Rate and Cost to Company of Long Term Issues Offered Publicly, by Quarters

1938		1939								1940							
4th Q		1st Q		2nd Q		3rd Q		4th Q		1st Q		2nd Q		3rd Q		4th Q	
Coupon rate	Cost to co.	Coupon rate	Cost to co.	Coupon rate	Cost to co.	Coupon rate	Cost to co.	Coupon rate	Cost to co.	Coupon rate	Cost to co.	Coupon rate	Cost to co.	Coupon rate	Cost to co.	Coupon rate	Cost to co.
(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)
3.50	3.47	4.00	4.13	3.50	3.65	4.00	4.14	3.50	3.55	2.75	2.69	3.25	3.41	4.00	4.05	3.50	3.23
3.25	3.30	3.75	3.91	3.75	4.01	4.00	3.69	4.00	4.04	3.50	3.43	3.50	3.56	3.00	2.85	3.25	3.01
3.50	3.45	3.50	3.59	4.00	4.19	4.00	4.12	4.00	4.10	3.75	3.71	3.00	2.92	3.25	3.17	4.00	4.07
3.75	3.85	3.50	3.32	4.50	4.88	3.00	2.77	3.75	3.90	3.25	3.07	3.50	3.51	4.00	4.11	3.50	3.17
3.50	3.45	4.00	4.22	4.50	3.70	4.00	4.05	3.25	3.20	4.00	4.02			4.50	4.80	3.00	2.88
4.00	4.41	3.50	3.93	5.50	6.36	3.75	3.85	4.00	4.04	3.75	3.14			4.00	4.00	4.00	4.33
3.50	3.64			3.75	3.88	3.75	3.84	3.75	3.79	3.00	2.91			4.00	4.44	3.25	3.02
3.25	3.15			3.00	2.89	3.50	3.34			3.50	3.40			3.25	3.19	4.50	5.06
3.75	3.82			3.50	3.27	4.50	4.44			4.50	4.63					2.75	2.62
3.50	3.41			3.75	3.84	3.75	3.72			4.00	4.02					3.00	2.74
4.00	4.14			3.25	3.12	4.00	4.36			3.50	3.48					3.25	3.09
6.00	6.91			3.50	3.36					3.25	3.22						
				3.75	3.79												

Source: *Journal of Land and Public Utility Economics*

(d) If the test performed in part (c) indicates that the two samples were drawn from the same parent population, combine the two coefficients into a single weighted coefficient.

12. From data collected on long-term bond issues it is desired to ascertain whether the correlations between coupon rates and costs of issue to the companies differ significantly when account is taken of the different dates at which the securities are issued. The accompanying table gives the material for such a study based on small samples.

(a) Compute separate correlation coefficients between coupon rates and costs of issue for the years 1939 and 1940.

(b) Transform the coefficients to corresponding z's (Table IV of Appendix B) and test to determine whether there is a significant difference in the coefficients for the two years. If the difference is found to be not significant, combine the coefficients into a single weighted coefficient.

13. Repeat the study outlined in Q12, using correlation coefficients calculated from the data for the first-half of 1939 and the first-half of 1940.

R. ANALYSIS OF VARIANCE

COMPARISON OF MEASURES OF VARIABILITY

1. The following table based on measurements of 868,445 draft recruits of the first World War gives the mean height, standard deviation of heights and the number of recruits measured in certain states of the United States.

State	Number of Men Measured	Mean Height (in inches)	Standard Deviation (in inches)
California	2,108	68.21	2.53
Kentucky	11,469	67.95	2.62
Minnesota	6,461	68.44	2.54
Nebraska	7,629	68.02	2.69
North Carolina	2,738	68.67	2.55
Utah	1,224	68.16	2.64
South Carolina	1,564	68.19	2.83
Wisconsin	3,296	68.13	2.66
Total U. S.	868,445	67.49	2.71

Source: The Medical Department of the United States Army in the World War, Volume XV, Statistics, Part I

(a) Determine whether the recruits from North Carolina and South Carolina differ significantly in variability, using the z-test developed by R. A. Fisher.

(b) Check the conclusion arrived at in (a) by calculating F and comparing it with the F values in table VIII of Appendix B for 1 per cent and 5 per cent standards of significance.

2. From material in problem R1, test the difference in variability of heights of recruits from Nebraska and Kentucky for significance,

(a) applying the z-test.

(b) computing F and comparing with Table VIII of Appendix B.

3. From material in problem R1, test the difference in variability of heights of recruits from California and Minnesota for significance,

(a) applying the z-test.

(b) computing F and comparing with table VIII of Appendix B.

4. From material in problem R1, test the difference in variability of heights of recruits from two states designated by the instructor for significance,

(a) applying the z-test.

(b) computing F and comparing with table VIII of Appendix B.

5. The following tabulations are of the average minimum starting salaries of payroll clerks and bookkeeping machine operators in 24 cities of the United States for the year 1940.

Payroll Clerks				Bookkeeping Machine Operators			
$19.38	$17.37	$17.15	$23.56	$20.60	$20.90	$19.00	$21.79
19.00	24.25	25.00	25.21	23.00	20.00	23.00	18.46
18.00	29.43	17.32	20.73	17.50	26.73	20.24	20.73
23.38	20.60	25.44	17.16	20.36	23.02	24.23	17.12
25.50	25.20	22.98	28.87	18.00	20.09	25.42	19.96
26.00	18.27	28.38	15.01	22.50	19.11	18.01	18.01

Source: American Business, Feb. 1941, p. 34

(a) Test the difference in variability in salary for the two groups of workers by means of the z-test.

(b) Check the conclusion arrived at in (a) by calculating F and comparing it with the F values in table VIII of Appendix B for 1 per cent and 5 per cent standards of significance.

6. The following tabulations are of the average minimum starting salaries of file clerks and switchboard operators in 35 cities of the United States for the year 1940.

File Clerks				Switchboard Operators			
$15.75	$15.00	$15.75	$19.33	$18.18	$18.00	$15.75	$21.08
15.39	15.00	16.54	18.48	18.48	15.00	16.37	19.62
17.50	16.37	15.17	17.48	22.00	17.25	15.67	22.98
18.00	15.00	13.85	17.50	21.75	15.25	17.31	18.50
16.81	20.15	23.08	15.00	20.05	22.61	20.77	15.00
15.00	18.00	13.85	17.32	18.54	25.00	16.16	18.48
19.31	15.00	18.87	16.16	19.73	15.00	20.97	16.00
14.47	18.00	18.43	13.00	18.53	21.50	22.26	13.00
16.40	13.85	17.32		18.44	15.00	19.25	

Source: American Business, Jan. 1941, p. 43

(a) Test the difference in variability in salary for the two groups of workers by means of the z-test.

(b) Check the conclusion arrived at in (a) by calculating F and comparing it with the F values in Table VIII of Appendix B for 1 per cent and 5 per cent standards of significance.

The Testing of Variability Between Classes

7. The following tabulation gives the wholesale prices in 1938 of five selected groups of commodities expressed as relatives of their prices in 1926.

Farm Products		Dairy Products	Cereal Products	Fruits and Vegetables	Other Foods
81.1	49.3	64.5	77.7	80.7	64.9
71.7	66.1	65.0	85.3	67.3	78.5

72.5	63.6	63.2	83.8	72.3	92.3
69.4	68.0	63.8	91.3	65.3	65.5
60.6	54.1	64.1	89.6	60.9	76.6
50.4	63.4	75.5	86.2	85.3	65.6
51.9	74.5	63.0	96.1	62.7	89.0
55.8	81.8	61.7	78.6	46.8	75.7
59.2	81.6	62.8	67.9	51.4	85.8
51.0	53.1	62.8	59.6	46.2	91.5
50.5	67.9	60.6	60.2	59.3	60.2
82.1	71.9	63.0	63.0	82.2	72.6
103.9	51.3	62.8	61.1	87.6	77.4
103.7	75.7	65.2	65.8	84.6	98.4
100.1	61.8	62.5	54.1	69.3	71.3
108.0	60.4	64.8	75.3	79.2	81.9
67.0	62.5	65.3	77.7	87.1	67.5
66.4	81.6	63.6	75.3	66.1	64.0
56.8	84.1	65.7	68.0	75.4	79.0
59.9	67.7	64.7	53.4		69.7
57.8	108.3	75.8			66.7
73.1	82.5				97.4
64.8	44.2				
49.4	107.4				
51.5	60.3				

Source: Wholesale Prices, 1938, U. S. Bureau of Labor Statistics

For the various classifications of commodities it is desired to apply an objective test to determine whether the observed differences in change of price are to be attributed to chance or whether the prices of these several classes were subject to the play of quite different forces during the period from 1926 to 1938.

(a) Using the data for dairy products, cereal products and for other foods, determine the variance within and between classes.

(b) Compute the value of z and by reference to the tabulation of values of z (Tables VI and VII of Appendix B) determine whether the results of the test are consistent with the hypothesis that the true value of z is zero.

(c) Check the conclusion arrived at in (b) by calculating F and comparing it with the F values (Table VIII of Appendix B) for 1 per cent and 5 per cent levels of significance.

8. Repeat the instructions of problem R7 above in a test of the differences in price movements for farm products, dairy products and fruits and vegetables.

9. Repeat the instructions of problem R7 above in a test of the differences in price movements for any three of the classes selected by the instructor.

VARIANCE ANALYSIS IN THE MEASUREMENT OF RELATIONSHIP

10. The following correlation table gives the total assets and total taxes (all kinds) of 100 United States corporations having assets under $70,000,000 for the fiscal year 1939.

Taxes in millions of dollars	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45–50	50–55	55–60	60–65	65–70
3–4						1			1	2	1		1	
2–3			1	1		1	2	2		3	1	3	2	1
1–2			2	4	5	2	6	1	4	1	4	1		
0–1	2	16	10	7	3	2	3	1	1	1		1		

Assets in millions of dollars

Source: Survey of Taxes for 1939, American Federation of Investors, Inc.

(a) Test the above data for the existence of correlation.

(b) Test the hypothesis of a linear relationship.

(c) If this hypothesis of linear relationship as tested in (*b*) is not to be maintained, test the hypothesis of a curvilinear relationship (using a power curve of the second degree, $y=a+bx+cx^2$).

11. The following correlation table shows the relation in 1935 between an index of real labor costs and an index of average selling prices based on a study of 90 industries in the United Kingdom. The base of both index series is 1930.

Labor Cost	40–60	60–80	80–100	100–120	120–140	140–160
140–160						1
120–140			1	1	1	1
100–120		3	7	6	1	
80–100		6	24	6		
60–80	1	15	11	1		
40–60	4					

Selling Price

Source: The Review of Economic Statistics; Vol. XXII, No. 2, May, 1940

(a) Test the above data for the existence of correlation.

(b) Test the hypothesis of a linear relationship.

(c) If the hypothesis of linear relationship as tested in (*b*) is not to be maintained, test the hypothesis of a curvilinear relationship (using a power curve of the second degree, $y=a+bx+cx^2$).

TESTING THE SIGNIFICANCE OF SEASONAL FLUCTUATIONS

12. The entries in the table on page 108 are the figures obtained when measurements of United States gasoline production by months, for the period 1927–1936, are expressed as percentages of linear trend ($y=25.403+.143x$, January 15, 1927, origin). Determine whether or not the seasonal fluctuations observed in the series represent a true seasonal pattern, by the application of variance analysis involving the following computations:

(a) Total variance.

Using an assumed mean of zero, this computation can be made readily from the expression $\Sigma d^2 = \Sigma(d')^2 - Nc^2$, when

d = deviation of an observation from the true mean,
d' = deviation from an assumed mean,
N = number of entries in the study,
c = difference between true and assumed mean (in this case being equal to the true mean)

(b) Variance between months.

Again, using an assumed mean of zero, this computation can be made readily from the expression $\Sigma d^2 = \Sigma(d')^2 - Nc^2$, when

d' = mean value for a given month, each squared value being weighted by the number of items represented by the mean.

(c) Variance within months.

Total variance minus variance between months.

(d) Variance within months due to factors affecting yearly conditions as a whole (cyclical and other forces).

Again, using an assumed mean of zero, this computation can be made readily from the expression $\Sigma d^2 = \Sigma(d')^2 - Nc^2$, when

d' = yearly average, each squared value being weighted by the number of months in a year.

(e) Variance within months corrected for factors affecting yearly conditions as a whole; residual variability due to chance.

Variance within months (computed in (*c*)) minus variance within months due to factors affecting yearly conditions (computed in (*d*)). (This computation may be made most readily by subtracting from the total variance determined in (*a*), the variance determined in (*b*) and (*d*).)

A convenient tabulation for the test of significance is as follows:

(1) Nature of Variability	(2) No. of Degrees of Freedom (n)	(3) Sum of Squares	(4) Mean Square (variance) σ^2 (3) ÷ (2)	(5) Natural Logarithm of Mean Square $\log_e \sigma^2$
Between means of years	– – –	– – – –		
Between means of months	– – –	– – – –	– – – – – –	– – – – – –
Residual variability	– – –	– – – –	– – – – – –	– – – – – –
Total	– – –	– – – –	Difference =	– – – – – –

$$z = \frac{\text{Difference}}{2} =$$

Refinery Production of Gasoline in the United States, 1927–1936

Monthly values as percentages of trend, with computations required in the analysis of variance

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Sum	Mean	Sum of Squares
1927	109.84	97.89	108.41	102.87	103.08	101.34	106.78	106.10	104.37	108.21	106.63	106.08	1,261.60	105.13	132,762.15
1928	103.10	98.70	106.93	106.35	111.38	110.64	117.80	120.70	119.14	121.15	115.83	118.26	1,349.98	112.50	152,481.35
1929	118.93	108.00	118.92	118.42	123.39	120.51	127.38	129.37	124.22	132.40	123.42	122.99	1,467.95	122.33	180,003.03
1930	117.12	108.21	119.62	120.37	123.78	116.86	118.56	117.56	115.18	118.39	102.78	105.32	1,383.75	115.31	160,017.59
1931	99.65	95.09	106.64	110.55	116.99	111.04	114.31	115.51	110.81	113.50	105.91	105.08	1,305.08	108.76	142,411.35
1932	96.62	91.53	94.51	97.50	101.65	97.63	95.49	94.00	87.99	94.16	90.57	87.88	1,128.53	94.13	106,504.39
1933	85.35	77.23	87.85	88.46	94.57	96.98	100.30	99.78	99.29	97.21	88.58	84.71	1,100.31	91.69	101,487.09
1934	89.07	81.50	87.14	90.49	91.42	90.87	96.37	97.11	89.45	93.72	91.63	92.40	1,091.17	90.93	99,408.60
1935	90.31	83.32	89.70	87.84	94.71	95.88	101.78	100.92	98.88	103.84	99.26	99.90	1,146.34	95.53	109,959.61
1936	96.82	90.73	94.24	96.68	101.30	100.12	104.32	106.53	104.86	108.92	102.06	105.31	1,211.89	100.99	122,716.87
Sum	1,006.81	932.20	1,013.96	1,019.53	1,062.27	1,041.87	1,083.09	1,087.58	1,054.19	1,091.50	1,026.67	1,027.93	12,477.60		1,307,752.03
Men	100.68	93.22	101.40	101.95	106.23	104.19	108.31	108.76	105.42	109.15	102.67	102.79		103.73	

(f) To determine whether or not the differences between the yearly averages are significant, the *z* test should be applied to the variance between means of years (as computed in (*d*), divided by the number of degrees of freedom) and the residual variance (computed in (*e*)).

13. Following the procedure outlined in problem 12, test the significance of seasonal fluctuations of monthly imports into the United States for the period 1931–1940. The figures obtained when import values by months are expressed as percentages of linear trend ($y = 75.2529 - .2058x$, origin at January 15, 1931), are given in the table on the next page.

14. Following the procedure outlined in problem 12, test the significance of seasonal fluctuations of boot and shoe production in the United States for the period 1925–1934. The figures obtained when production data by months are expressed as percentages of centered 12-month's moving averages are given in the table on page 110. Note that the use of moving averages instead of a linear trend as the base for the percentage calculations obviates steps (*d*) and (*e*) as outlined in problem 1.

FURTHER APPLICATION

15. The following tabulation shows the retail price of bituminous coal on December 15, 1938, expressed as a per cent of the price on March 15, 1937, by grade and size of coal, in various cities of the United States.

Grade of coal	High Volatile Eastern			High Volatile Western			Low Volatile		
Size of Coal	Lump	Egg	Stoker	Lump	Egg	Stoker	Lump	Egg	Stoker
	97.8	99.7	103.4	99.7	99.8	102.1	98.7	103.0	111.5
	97.8	105.3	98.0	100.2	101.9	107.8	104.7	102.6	105.5
	94.5	105.6	102.8	98.5	99.0	110.3	101.1	101.8	103.8
	98.8	100.0	102.5	100.0	104.8	101.4	100.8	101.2	108.4
	98.6	106.3	99.6	97.8	98.7	111.8	99.4	104.6	104.1
	96.8	104.0	95.7	98.8	101.2	113.1	104.5	112.5	107.2
	98.5	106.9	103.3	100.0	105.4	108.4	101.7	106.5	105.5
	98.7	100.3	100.4	101.4	105.4	112.7	100.0	102.6	106.7
	98.3		103.6	99.1		104.7	100.0		103.5
	93.6		98.2	100.0		112.6	101.3		105.4

Source: Original data from *Retail Prices*, U. S. Bureau of Labor Statistics, January, 1939.

(a) Test the significance of the variability between high volatile eastern and low volatile grades of coal subdivided into
  (1) lump and stoker sizes,
  (2) lump and egg sizes,
  (3) any two sizes suggested by the instructor,
  (4) lump, egg and stoker sizes.

(b) Test the significance of the variability between high volatile eastern and high volatile western grades of coal subdivided into one of the classifications listed in (a).

(c) Test the significance of the variability between high volatile western and low volatile grades of coal subdivided into one of the classifications listed in (a).

Value of Imports into the United States, 1931–1940

Monthly values as percentages of trend, with computations required in the analysis of variance

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Sum	Mean	Sum of Squares
1931	75.7	72.0	86.9	77.7	75.2	72.8	73.0	70.5	72.0	70.8	62.9	64.4	873.9	72.83	64,057.93
1932	57.7	56.5	56.7	54.0	48.6	47.4	34.9	39.2	42.2	46.5	45.2	42.5	571.4	47.62	27,808.18
1933	42.7	37.1	41.5	38.7	47.5	54.8	63.7	69.7	65.5	68.7	58.6	60.3	648.8	54.07	36,674.70
1934	61.9	60.6	72.7	66.9	71.6	62.9	58.5	55.7	61.9	60.6	71.4	62.5	767.2	63.93	49,380.76
1935	79.5	72.1	84.7	81.8	82.1	76.2	85.7	81.3	78.5	92.9	82.1	91.9	988.8	82.40	81,863.70
1936	92.2	95.7	99.2	101.1	95.0	95.4	97.3	97.6	109.4	108.1	100.3	125.3	1,216.6	101.38	124,254.54
1937	122.4	142.8	158.3	148.8	147.6	149.8	138.5	128.8	122.5	117.8	118.2	111.7	1,607.2	133.93	217,902.64
1938	91.4	86.6	93.8	87.2	80.5	79.0	77.5	90.2	92.3	98.0	98.4	95.1	1,070.0	89.17	95,969.20
1939	99.1	88.6	107.1	105.7	115.2	101.0	95.8	99.9	104.0	124.9	136.6	142.7	1,320.6	110.05	148,382.22
1940	141.4	117.3	127.3	125.9	124.4	125.0	138.9	131.8	116.7	125.0	135.3	153.6	1,562.6	130.22	204,717.10
Sum	864.0	829.3	928.2	887.8	887.7	864.3	863.8	864.7	865.0	913.3	909.0	950.0	10,627.1		1,051,010.97
Mean	86.40	82.93	92.82	88.78	88.77	86.43	86.38	86.47	86.50	91.33	90.90	95.00		88.56	

Boot and Shoe Production in the United States, 1925–1934

Monthly values as percentages of trend and cycle, with computations required in the analysis of variance

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Sum	Mean	Sum of Squares
1925	98.5	99.3	111.2	109.3	93.0	86.7	92.2	106.3	111.6	116.9	93.2	92.4	1,210.6	100.833	123,181.66
1926	90.2	97.0	112.4	99.3	85.9	92.6	92.6	108.8	116.1	115.3	97.1	91.4	1,198.7	99.892	120,960.73
1927	89.0	96.1	109.1	98.6	88.9	95.8	96.9	121.9	116.9	111.8	90.0	81.3	1,196.3	99.692	120,985.59
1928	90.7	102.4	112.2	92.7	91.7	94.8	98.3	122.0	108.4	116.8	91.7	75.5	1,197.2	99.767	121,339.14
1929	93.5	94.5	104.7	98.7	97.3	93.4	100.3	121.3	116.8	125.3	93.9	77.3	1,217.0	101.417	125,475.74
1930	92.3	92.2	104.0	107.8	93.9	93.7	96.4	115.0	119.1	112.1	74.3	69.2	1,170.0	97.500	116,632.18
1931	77.7	92.3	111.8	113.7	108.8	105.7	108.3	125.9	117.2	95.5	70.6	76.3	1,203.8	100.317	124,235.28
1932	84.1	104.8	123.8	102.8	87.2	90.4	77.9	117.6	129.4	126.3	94.0	72.8	1,211.1	100.925	126,456.19
1933	79.4	89.8	96.6	93.9	112.6	119.5	115.0	125.0	103.7	102.6	76.5	65.3	1,179.9	98.325	119,682.97
1934	85.8	101.7	119.5	116.2	115.6	96.3	95.0	118.3	94.0	95.7	79.9	78.1	1,196.1	99.675	121,621.27
Sum	881.2	970.1	1,105.3	1,033.0	974.9	968.9	972.9	1,182.1	1,133.2	1,118.3	861.2	779.6	11,980.7		1,220,570.75
Mean	88.12	97.01	110.53	103.30	97.49	96.89	97.29	118.21	113.32	111.83	86.12	77.96		99.839	

APPENDIX A

PROBLEM DATA

TABLE 1

Selected Time Series for the United States (monthly averages)

Year	Steel Ingot Production (million short tons)	Electric Power Production (billion kilowatt hours)	Automobile Production (factory sales) (thousand cars)	Index of Textile Manufactures (F. R. Bd. 1935–39 = 100)	Index of Petroleum Refining (F. R. Bd. 1935–39 = 100)	Index of Manufacturing Employment (U. S. Dept. of Labor, 1923–1925 = 100)
1920	3.84	3.61	185.6	66	34	107.1
1921	1.80	3.41	134.7	69	34	82.0
1922	3.24	3.96	212.0	78	40	90.7
1923	4.08	4.63	336.2	83	46	103.8
1924	3.45	4.91	300.2	72	53	96.4
1925	4.14	5.48	355.5	84	62	99.8
1926	4.41	6.14	358.4	84	68	101.7
1927	4.11	6.61	283.4	92	72	99.5
1928	4.72	7.21	363.2	87	81	99.7
1929	5.15	7.99	446.5	94	90	106.0
1930	3.72	7.89	279.7	74	85	92.4
1931	2.38	7.56	199.1	79	80	78.1
1932	1.26	6.87	114.2	71	72	66.3
1933	2.14	7.06	160.0	88	75	73.4
1934	2.43	7.57	229.4	76	79	85.7
1935	3.18	8.21	328.9	93	86	91.3
1936	4.46	9.35	371.2	104	95	99.0
1937	4.72	10.15	400.7	106	106	108.6
1938	2.65	9.72	207.4	85	103	90.9
1939	4.40	10.86	298.1	112	111	99.9
1940	5.58	12.08	372.4	115	113	107.5

Year	Index of Construction Contracts Awarded (F. R. Bd. 1923–25 = 100)	Index of Construction Costs (Eng. News Record, 1913 = 100)	Total Freight Car Loadings (million cars)	Total U. S. Bank Debits (billion dollars)	Index of Department Store Sales (F. R. Bd. 1923–25 = 100)	Stock Sales on New York Stock Exchange (million shares)	U.S. D. of L. Wholesale Commodity Price Index (1926 = 100)
1920	63	253.3	3.76	40.25	94	18.73	154.4
1921	56	196.6	3.28	33.25	87	14.24	97.6
1922	79	176.4	3.60	36.61	88	21.73	96.7
1923	84	216.3	4.15	38.64	98	19.77	100.6
1924	94	214.7	4.05	40.97	99	23.50	98.1
1925	122	206.4	4.27	47.51	103	37.68	103.5
1926	129	208.4	4.43	50.66	106	37.43	100.0
1927	129	205.6	4.30	56.16	107	48.08	95.4
1928	135	207.2	4.30	67.20	108	76.71	96.7
1929	117	207.0	4.40	77.92	111	93.75	95.3
1930	92	201.6	3.82	55.16	102	67.55	86.4
1931	63	178.7	3.10	40.11	92	48.08	73.0
1932	28	156.6	2.35	26.86	69	35.44	64.8
1933	25	172.9	2.44	25.70	67	54.57	65.9
1934	32	198.7	2.57	27.63	75	26.99	74.9
1935	37	196.7	2.63	31.18	79	31.81	80.0
1936	55	208.1	3.01	35.72	88	41.34	80.8
1937	59	236.3	3.14	36.09	92	34.12	86.3
1938	64	235.4	2.54	31.13	85	24.79	78.6
1939	72	235.8	2.84	32.47	90	21.84	77.1
1940	81	242.9	3.03	33.96	94	17.30	78.5

Source: Survey of Current Business

TABLE 2

Selected Time Series for the United States

Year	Total U. S. Pig Iron Production (1,000,000 short tons)	Total U. S. Bituminous Coal Production (1,000,000 short tons)	Total U. S. Coke Production (1,000,000 short tons)	Total U. S. Steel Production (1,000,000 short tons)	Total U. S. Petroleum Production (1,000,000 barrels)	Total U. S. Cement Production (1,000,000 barrels)	Total U. S. Cotton Consumption (1,000,000 bales)
1900	11.12	—	20.53	8.22	63.62	17.23	3.69
1901	12.81	225.83	21.80	10.86	69.39	20.07	3.60
1902	14.37	260.22	25.40	12.06	88.76	25.75	4.08
1903	14.52	282.75	25.27	11.72	100.46	29.90	4.19
1904	13.31	278.66	23.66	11.18	117.08	31.68	3.98
1905	18.54	315.06	32.25	16.15	134.72	40.10	4.52
1906	20.41	342.88	36.40	18.87	126.49	51.00	4.91
1907	20.79	394.76	40.78	18.84	166.10	52.23	4.99
1908	12.85	332.57	26.04	11.31	178.53	52.91	4.54
1909	20.81	379.74	39.32	19.32	183.17	66.69	5.09
1910	22.02	417.11	41.71	21.04	209.56	77.79	4.62
1911	19.07	405.91	35.55	19.10	220.45	79.55	4.50
1912	23.98	450.11	43.98	25.20	222.94	83.35	5.13
1913	34.41	478.44	46.30	25.24	248.45	92.10	5.58
1914	25.82	422.70	34.56	18.96	265.76	88.24	5.45
1915	33.22	442.62	41.58	25.93	281.10	85.75	6.01
1916	43.73	502.52	54.53	34.49	300.77	91.07	6.62
1917	42.77	551.79	55.61	49.65	335.32	92.65	6.82
1918	43.13	579.39	56.48	48.88	355.93	70.69	6.18
1919	34.25	465.86	44.18	38.03	378.37	80.40	5.92
1920	40.78	568.67	51.35	46.10	442.93	99.67	5.84
1921	18.53	415.92	25.30	21.63	472.18	98.29	5.41
1922	30.11	422.27	37.12	38.91	557.53	113.87	6.09
1923	44.87	564.57	56.98	48.97	732.41	137.38	6.52
1924	34.84	483.69	44.27	41.42	713.94	148.86	5.52
1925	40.77	520.05	51.26	49.68	763.74	161.21	6.43
1926	43.76	573.37	56.87	52.89	770.87	164.08	6.68
1927	40.58	517.36	51.10	49.26	901.13	171.91	7.41
1928	42.38	500.75	52.80	56.62	1,007.32	175.97	6.57
1929	47.36	534.99	59.88	61.74	898.01	170.20	7.05
1930	35.17	467.53	47.96	44.59	851.08	160.91	5.38
1931	20.47	382.09	33.48	28.61	785.16	124.57	5.44
1932	9.73	309.71	21.78	15.12	905.66	76.51	5.02
1933	14.80	333.63	27.59	25.72	908.07	63.37	6.21
1934	17.82	359.37	31.82	29.18	996.60	77.69	5.42
1935	23.53	372.37	35.14	38.18	1,099.69	76.48	5.65
1936	34.29	439.09	46.27	53.50	1,279.16	112.37	7.10
1937	41.01	445.53	52.38	56.64	1,213.25	116.48	7.42
1938	21.04	348.55	32.66	31.75	1,214.35	105.55	5.90
1939	35.32	389.52	44.42	52.80	1,264.26	121.82	7.37
1940	46.95	453.22	56.75	66.99	1,361.85	130.29	8.06

Source: Statistical Abstract of the United States

Table 3

Price Relatives at Wholesale for 195 Commodities for 1897
(1891 = 100)

82.6	72.3	79.9	79.9	100.5	76.8
53.0	68.7	89.2	95.5	98.4	83.6
44.4	86.4	81.1	101.7	88.3	77.1
47.3	86.9	66.4	60.6	100.0	61.4
47.5	93.9	84.0	65.3	95.4	86.4
80.5	91.7	100.0	68.6	82.4	69.1
68.7	90.8	106.3	67.7	88.6	63.5
43.9	83.9	80.0	70.7	72.3	65.6
92.3	78.8	81.6	84.3	109.4	86.3
88.8	76.5	81.5	78.3	76.8	81.8
93.6	77.1	80.0	85.1	59.8	67.0
81.2	73.4	76.8	85.7	79.4	82.4
85.8	70.6	86.6	83.1	78.8	82.3
82.4	73.6	93.1	86.5	86.5	70.4
77.5	96.0	80.8	64.8	38.5	102.8
95.2	66.7	88.4	69.0	46.4	67.8
42.5	93.8	76.3	94.3	99.4	85.1
46.4	57.0	76.7	108.1	110.9	46.9
84.4	88.9	65.9	101.3	83.3	68.7
55.0	86.5	79.7	104.2	83.1	120.5
77.8	100.0	67.7	72.2	90.5	72.9
111.6	100.0	70.0	87.7	117.3	72.9
47.3	55.3	66.7	80.6	111.1	71.4
75.8	52.3	67.4	117.5	106.3	92.0
99.3	50.9	91.3	86.2	68.4	83.3
83.7	93.9	86.6	100.0	64.7	93.8
100.0	72.2	92.7	87.0	59.5	64.5
79.6	95.8	83.1	102.3	62.7	85.7
91.0	67.2	100.0	106.8	55.9	79.8
105.3	77.2	81.0	93.1	60.2	83.3
80.0	83.9	94.2	83.7	86.2	75.7
85.0	95.2	109.1	69.8	90.5	84.2
83.3	82.1	100.0			

Source: Mills, F. C., *The Behavior of Prices*

TABLE 4

Price Relatives at Wholesale for 391 Commodities for the Year 1923 (1913=100)

224.9	133.2	197.7	76.9	244.5	143.3
224.5	163.5	114.5	200.0	231.3	164.2
153.2	156.7	114.4	206.0	207.3	168.9
149.7	160.7	103.3	336.4	210.9	166.1
189.9	169.2	162.8	120.0	236.1	152.1
261.8	150.7	136.6	154.2	257.1	156.4
213.5	111.5	139.1	107.3	232.4	175.0
181.0	180.2	150.7	125.1	207.1	181.1
108.7	132.1	152.4	345.5	204.5	166.8
135.5	153.5	75.7	139.1	209.9	258.1
126.6	160.4	155.3	187.3	174.8	168.9
178.8	139.3	104.3	235.6	182.7	160.4
124.8	178.7	158.0	146.7	240.1	104.6
122.1	174.7	159.8	115.5	193.4	104.4
116.1	162.5	128.8	112.9	164.4	175.6
83.9	173.5	110.6	189.8	147.6	106.5
170.3	104.2	114.3	86.4	168.9	118.0
178.0	127.2	242.3	110.6	237.2	220.1
127.3	123.4	192.0	87.5	115.8	158.5
116.2	136.1	227.6	189.0	220.7	212.6
120.8	155.8	150.3	116.4	219.9	213.5
124.9	129.5	189.7	112.6	188.0	124.6
117.1	151.4	152.1	233.4	162.7	159.7
91.8	170.7	173.2	198.8	156.2	236.1
94.0	191.1	221.1	227.7	161.5	128.9
149.6	190.4	171.1	140.2	160.0	220.0
145.2	103.7	166.9	130.2	152.2	182.0
149.0	145.9	188.1	140.5	174.0	185.1
151.0	183.7	107.6	175.9	188.5	172.7
132.8	109.9	73.0	122.8	153.6	315.8
164.8	202.9	194.4	182.1	175.9	130.3
275.4	168.0	176.1	219.4	203.8	131.4
196.9	122.6	153.9	165.9	198.7	131.8
252.5	148.4	126.6	222.2	168.7	116.8
148.1	120.7	133.6	234.5	107.0	118.2
163.7	90.3	63.6	230.3	92.2	116.9
220.5	169.0	124.5	150.9	168.0	200.5
296.9	75.2	206.4	176.7	95.0	158.1
191.0	50.2	209.3	190.8	120.2	241.8
226.6	200.3	141.7	234.5	154.6	204.8
236.8	123.0	233.4	195.6	192.3	214.6
226.0	117.0	201.3	222.6	163.3	214.9
228.9	91.9	215.7	201.5	139.3	195.8
95.0	92.7	216.8	243.3	150.2	237.1
90.9	130.2	205.8	173.9	134.7	202.4
153.5	172.5	195.6	177.6	124.7	196.2
83.4	143.0	255.3	123.0	242.5	210.0
71.2	134.8	193.9	273.7	240.3	164.9
82.4	152.3	207.4	177.2	148.4	221.4
90.6	202.3	199.8	121.6	152.2	140.6
130.1	142.4	207.3	114.1	155.1	228.3
223.0	170.7	212.4	112.8	148.3	168.5
204.4	189.0	228.6	87.4	152.8	299.5
233.1	178.6	220.5	214.9	125.3	183.3
226.1	157.4	237.7	113.6	149.2	146.6
212.4	116.9	181.9	164.4	153.0	159.8
30.8	148.4	148.1	113.8	152.7	175.5
101.7	112.8	221.0	101.6	146.2	169.1
90.1	103.1	201.4	122.4	147.2	120.5
153.8	99.0	184.3	113.3	145.8	181.7
126.5	139.3	242.6	122.6	153.7	166.5
105.5	139.2	210.9	274.6	155.7	173.2
118.7	145.1	290.7	217.4	192.5	143.4
126.8	153.3	216.4	124.3	127.1	153.2
135.2	171.6	148.7	171.6	120.2	140.8
					140.0

Source: Mills, F. C., *The Behavior of Prices*

Table 5

Age Distribution of Women Factory Workers in 24 New England Plants in 1937 and 1938

Age Groups	Number of Women Workers 1937	1938
15 to 19 years	956	400
20 to 24 years	3,172	1,825
25 to 29 years	1,901	1,399
30 to 34 years	1,003	846
35 to 39 years	870	706
40 to 44 years	669	566
45 to 49 years	430	373
50 to 54 years	277	273
55 to 59 years	143	140
60 to 64 years	86	85
65 to 69 years	38	40
70 to 74 years	10	7
	9,555	6,661

Source: Monthly Labor Review, April 1939, p. 769

Table 6

Degree of Price Increase of 149 Commodities during Ten Periods of Revival and Prosperity in the United States, Based on 1,110 Observations

Degree of Rise (as a percentage of the ensuing high)	Frequency
.50 to 6.49	68
6.50 to 12.49	100
12.50 to 18.49	126
18.50 to 24.49	139
24.50 to 30.49	131
30.50 to 36.49	126
36.50 to 42.49	92
42.50 to 48.49	93
48.50 to 54.49	69
54.50 to 60.49	52
60.50 to 66.49	40
66.50 to 72.49	33
72.50 to 78.49	30
78.50 to 84.49	10
84.50 to 90.49	1
Total	1,110

Source: Mills, F. C., *The Behavior of Prices*, p. 414

Table 7

Average Annual Rates of Change in Price of 223 Commodities in the United States Between 1896 and 1913

Rate of Change in Price between 1896 and 1913	Number of Commodities
−3.50 to −2.51	4
−2.50 to −1.51	3
−1.50 to − .51	8
− .50 to + .49	31
+ .50 to +1.49	41
+1.50 to +2.49	58
+2.50 to +3.49	39
+3.50 to +4.49	20
+4.50 to +5.49	15
+5.50 to +6.49	2
+6.50 to +7.49	1
+7.50 to +8.49	0
+8.50 to +9.49	0
+9.50 to +10.49	1
Total	223

Source: Mills, F. C., *The Behavior of Prices*, p. 382

Table 8

Number of Wives Entering Widowhood According to the Age of Wife at the Death of Husband in the United States, 1936–1938

Age of Wife at Death of Husband	Estimated Number of Wives Entering Widowhood
Under 20	2,100
20 to 24	9,200
25 to 29	15,300
30 to 34	20,200
35 to 39	26,100
40 to 44	34,300
45 to 49	42,100
50 to 54	46,800
55 to 59	50,500
60 to 64	48,300
65 to 69	42,000
70 to 74	31,300
75 to 79	17,600
80 and over	9,000
Total	394,800

Source: Metropolitan Life Insurance Company, *Statistical Bulletin*, Volume 21, No. 8, August, 1940

TABLE 9

Year	Wheat Flour, Wholesale Price, Winter, Straight (Kansas City) (mo. av.) (dollars per barrel)	Price Received by Farmers for Raw Cotton (mo. av.) (dollars per pound)
1921	$7.03	$.125
1922	6.13	.193
1923	5.35	.270
1924	5.98	.268
1925	7.68	.222
1926	7.25	.151
1927	6.69	.159
1928	6.41	.186
1929	5.79	.178
1930	4.87	.124
1931	3.57	.076
1932	3.10	.058
1933	4.63	.081
1934	5.76	.120
1935	6.20	.116
1936	5.44	.118
1937	5.61	.111
1938	4.36	.083
1939	3.87	.087
1940	4.31	.096

Source: Survey of Current Business

TABLE 10

Average Monthly Price of Selected Grains in United States Markets, 1925–1940
(dollars per bushel)

Year	Corn	Oats	Rice	Rye	Wheat	Barley
1925	$.97	$.45	$2.93	$1.09	$1.63	$.76
1926	.70	.41	2.79	.92	1.49	.62
1927	.80	.47	1.85	1.00	1.35	.76
1928	.92	.54	1.71	1.07	1.26	.77
1929	.90	.47	1.71	.96	1.18	.61
1930	.79	.39	1.80	.61	.92	.51
1931	.51	.27	1.40	.39	.64	.43
1932	.31	.20	.99	.37	.55	.37
1933	.39	.28	1.26	.56	.75	.46
1934	.65	.44	1.76	.71	1.01	.82
1935	.80	.40	1.80	.55	1.04	.71
1936	.81	.36	1.80	.71	1.14	.86
1937	1.01	.42	1.62	.92	1.21	.89
1938	.53	.29	1.49	.54	.79	.48
1939	.50	.34	1.53	.49	.79	.59
1940	.63	.37	1.71	.55	.89	.49

Grades quoted are: Corn, wholesale, weighted average, 5 markets, all grades; Barley, wholesale, #3 straight, Minneapolis; Rice, wholesale, head, clean, New Orleans; Rye, wholesale, #2, Minneapolis; Oats, wholesale, #3 white, Chicago; Wheat, wholesale, weighted average—6 markets, all grades.

Source: Survey of Current Business

Table 11

Annual Production of Selected Grains in United States, 1925–1940
Production (million bushels)

Year	Corn	Oats	Rice	Rye	Wheat	Barley
1925	2,798	1,405	33	42	669	192
1926	2,547	1,153	42	35	832	166
1927	2,616	1,093	44	51	875	239
1928	2,666	1,313	44	38	914	328
1929	2,521	1,113	40	35	823	280
1930	2,080	1,275	45	45	886	300
1931	2,576	1,124	45	33	942	199
1932	2,931	1,251	42	39	757	298
1933	2,400	733	38	21	552	154
1934	1,461	542	39	17	526	117
1935	2,304	1,195	39	59	626	286
1936	1,507	786	50	25	627	147
1937	2,651	1,162	53	50	876	220
1938	2,562	1,068	53	56	932	253
1939	2,619	937	52	39	755	276
1940	2,449	1,236	53	41	817	309

Source: Survey of Current Business

Table 12

Average Retail Prices of Selected Articles of Food in the United States (1923–1937) in Cents

	Bread (White) per Pound	Sirloin Steak per Pound	Butter per Pound	Fresh Milk per Quart	Eggs per Dozen
1923	8.8	40.0	55.8	13.9	49.9
1924	8.9	40.9	52.2	13.4	51.0
1925	9.3	42.4	55.2	13.9	55.4
1926	9.3	43.0	53.6	14.0	51.9
1927	9.2	44.8	56.3	14.1	48.7
1928	8.9	50.1	56.9	14.2	50.3
1929	8.8	52.1	55.5	14.4	52.7
1930	8.6	45.4	46.4	14.1	44.5
1931	7.7	41.0	35.8	12.6	35.0
1932	7.0	34.4	27.8	10.7	30.2
1933	7.1	29.6	27.8	10.4	28.8
1934	8.1	33.2	30.9	11.2	27.5
1935	8.3	42.4	31.4	11.8	34.6
1936	8.1	37.0	35.8	11.8	32.8
1937	8.7	44.0	38.3	12.1	32.5

Source: Retail Prices, U. S. Bureau of Labor Statistics

Apparent per capita consumption of selected food products in the United States for the years 1926–1928:

Flour and cereals	**215 pounds**	**Milk**	**65 quarts**
Lean meat	**175 pounds**	**Eggs**	**18 dozen**
Fats and oils	**42 pounds**		

Source: Vaile & Canoyer: ***Income and Consumption,*** **p. 218**

TABLE 13

Prices and Apparent Consumption of Selected Food Products, (monthly averages), 1930–1940

Prices

Year	Eggs, Average Wholesale Price for Five Cities (per dozen)	White Potatoes, Wholesale, New York (dollars per 100 lbs.)	Beef and Veal, Wholesale, Fresh, Native Steers, Chicago (dollars per pound)	Butter, Wholesale, Creamery 92 Score, New York (dollars per pound)
1930	$.285	$2.373	$.207	$.37
1931	.220	1.523	.156	.28
1932	.196	1.083	.131	.21
1933	.177	1.677	.095	.22
1934	.209	1.535	.114	.26
1935	.270	1.060	.176	.30
1936	.258	2.222	.152	.33
1937	.242	1.649	.211	.34
1938	.242	1.256	.158	.28
1939	.194	1.628	.163	.26
1940	.204	1.787	.174	.30

Apparent Consumption

Year	Eggs, Receipts in Five Markets (million dozen)	White Potatoes, Production (million cwt.)	Beef and Veal, Apparent Consumption (million pounds)	Butter, Apparent Consumption (million pounds)
1930	40.4	17.0	398	134
1931	40.1	19.2	398	142
1932	34.4	18.8	368	142
1933	36.6	17.1	420	139
1934	34.1	20.3	464	147
1935	33.4	19.3	440	138
1936	35.4	16.6	497	135
1937	36.2	19.8	466	138
1938	33.9	18.7	454	142
1939	36.5	18.1	414	153
1940	37.9	19.9	464	152

Source: Survey of Current Business; Wholesale Prices, U. S. Bureau of Labor Statistics

TABLE 14

Year	Indicated Expenditures for Total Building Construction (based on building permits) (mo. av. 1929 = 100)
1929	69.8
1930	57.2
1931	40.4
1932	14.8
1933	12.2
1934	12.5
1935	21.4
1936	33.3
1937	37.0
1938	38.0
1939	46.9
1940	52.2

Source: U. S. Bureau of Labor Statistics

TABLE 15

Selected Time Series

Year and Month	U. S. Portland Cement Production (million bbls.)	U. S. Meat Consumption (million pounds)	U. S. Boot and Shoe Production (million pairs)	U. S. Cotton Consumption (thousand bales)	U. S. Exports (million dollars)	U. S. D. of L. Wholesale Commodity Price Index (1926 = 100)	U.S. Bank Debits (billion dollars)
1920							
Jan.	6.29	—	—	592	722	157.7	45.37
Feb.	6.09	—	—	516	645	157.1	35.79
Mar.	7.06	—	—	576	820	158.6	43.43
Apr.	7.76	—	—	567	684	165.5	41.64
May	8.87	—	—	541	746	167.2	39.26
June	8.85	—	—	555	629	166.5	40.35
July	8.54	—	—	526	651	165.8	39.87
Aug.	9.46	—	—	484	578	161.4	36.27
Sept.	9.47	—	—	458	605	155.3	37.38
Oct.	10.33	—	—	401	751	144.2	41.03
Nov.	9.43	—	—	333	677	133.4	39.70
Dec.	7.54	—	—	295	720	120.7	42.95
Ave.	8.31	—	—	487	686	154.4	40.25
1921							
Jan.	4.10	—	—	367	654	114.0	38.03
Feb.	4.38	—	—	395	487	104.9	29.73
Mar.	6.76	—	—	438	387	102.4	33.90
Apr.	8.65	—	—	409	341	98.9	32.24
May	9.28	—	—	441	330	96.2	32.51
June	9.30	—	—	462	337	93.4	33.61
July	9.57	—	—	410	325	93.4	31.51
Aug.	10.24	—	—	467	367	93.5	30.10
Sept.	10.03	—	—	485	325	93.4	31.63
Oct.	10.51	—	—	494	343	94.1	34.32
Nov.	8.92	—	—	528	294	94.2	33.44
Dec.	6.56	—	—	511	296	92.9	38.02
Ave.	8.19	—	—	451	374	97.6	33.25
1922							
Jan.	4.29	—	25.1	527	279	91.4	34.94
Feb.	4.28	—	24.5	472	251	92.9	30.59
Mar.	6.69	—	29.3	520	330	92.8	36.93
Apr.	9.24	—	26.8	444	319	93.2	36.39
May	11.18	—	26.2	495	308	96.1	37.98
June	11.25	—	24.8	509	335	96.3	39.24
July	11.56	—	22.7	458	301	99.4	36.06
Aug.	11.66	—	27.7	526	302	98.6	34.14
Sept.	11.42	—	28.3	494	313	99.3	35.77
Oct.	12.29	—	30.4	534	371	99.6	40.75
Nov.	11.35	—	30.1	579	380	100.3	36.16
Dec.	8.67	—	27.9	529	344	100.7	40.44
Ave.	9.49	—	27.0	507	319	96.7	36.61
1923							
Jan.	7.99	1,051	30.7	610	335	102.0	41.75
Feb.	8.21	899	30.4	567	307	103.3	35.93
Mar.	9.88	1,025	35.8	624	341	104.5	42.19
Apr.	11.36	931	31.8	577	326	103.9	39.29
May	12.91	1,093	36.9	621	316	101.9	40.07
June	12.38	1,034	28.3	542	320	100.3	40.57
July	12.62	1,035	25.3	463	302	98.4	36.50
Aug.	12.97	1,109	30.0	492	311	97.8	33.50
Sept.	13.11	1,043	27.6	486	381	99.7	34.06
Oct.	13.35	1,237	30.7	543	399	99.4	38.91
Nov.	12.60	1,107	27.0	533	402	98.4	38.50
Dec.	10.00	1,073	23.0	464	427	98.1	42.45
Ave.	11.45	1,053	29.8	543	347	100.6	38.64

TABLE 15—*Continued*

Year and Month	U. S. Portland Cement Production (million bbls.)	U. S. Meat Consumption (million pounds)	U. S. Boot and Shoe Production (million pairs)	U. S. Cotton Consumption (thousand bales)	U. S. Exports (million dollars)	U. S. D. of L. Wholesale Commodity Price Index (1926 = 100)	U. S. Bank Debits (billion dollars)
1924							
Jan.	8.79	1,152	26.5	578	395	99.6	41.50
Feb.	8.59	977	26.8	509	366	99.7	37.40
Mar.	10.37	939	28.9	486	340	98.5	40.74
Apr.	11.73	1,008	28.0	479	347	97.3	39.52
May	13.78	1,094	25.2	414	335	95.9	40.04
June	13.54	1,050	22.5	350	307	94.9	40.23
July	14.03	1,132	21.4	347	277	95.6	40.13
Aug.	15.13	1,022	25.5	357	331	97.0	38.69
Sept.	14.52	1,088	27.7	438	428	97.1	38.97
Oct.	14.82	1,227	30.8	534	527	98.2	43.42
Nov.	13.14	1,100	25.3	495	494	99.1	41.89
Dec.	10.44	1,150	24.6	534	446	101.5	49.11
Ave.	12.41	1,078	26.1	460	383	98.1	40.97
1925							
Jan.	8.86	1,146	26.1	594	446	102.9	49.98
Feb.	8.26	906	26.5	551	371	104.0	41.52
Mar.	11.03	941	29.9	583	454	104.2	47.62
Apr.	13.81	942	29.5	597	398	101.9	44.56
May	15.50	1,024	25.1	532	371	101.6	46.60
June	15.39	1,016	23.4	494	323	103.0	48.63
July	15.64	1,021	24.7	484	340	104.3	47.04
Aug.	15.42	978	28.5	451	380	103.9	43.13
Sept.	16.00	1,031	29.8	483	420	103.4	45.26
Oct.	15.99	1,198	31.1	544	491	103.6	52.96
Nov.	13.66	1,001	24.6	543	448	104.5	48.37
Dec.	10.71	1,090	24.4	576	468	103.4	54.40
Ave.	13.43	1,024	27.0	536	409	103.5	47.51
1926							
Jan.	7.89	1,072	23.9	582	397	103.2	54.15
Feb.	7.73	867	25.7	565	353	102.0	44.92
Mar.	10.39	1,016	30.0	636	374	100.6	56.46
Apr.	12.44	991	26.6	578	388	100.3	51.84
May	16.51	999	23.1	516	357	100.5	48.02
June	16.87	1,060	25.0	519	338	100.4	50.66
July	17.13	1,004	25.1	462	368	99.5	50.96
Aug.	17.00	1,004	29.6	500	384	99.1	47.01
Sept.	16.57	1,119	31.7	571	448	99.7	46.95
Oct.	16.60	1,122	31.7	508	455	99.4	52.54
Nov.	14.19	1,067	26.8	584	480	98.4	47.38
Dec.	10.76	1,069	25.4	603	465	97.9	57.07
Ave.	13.67	1,033	27.0	557	401	100.0	50.66
1927							
Jan.	8.26	1,059	25.0	603	419	96.5	54.71
Feb.	7.38	896	27.3	590	372	95.8	48.22
Mar.	11.45	1,034	31.3	693	409	94.7	58.52
Apr.	14.05	954	28.4	618	415	94.1	55.58
May	16.70	1,064	25.6	630	393	94.2	54.14
June	17.22	1,090	27.5	660	357	94.1	56.82
July	17.41	1,013	27.8	570	342	94.3	53.68
Aug.	18.32	1,114	35.1	635	375	95.2	53.70
Sept.	17.51	1,080	34.0	628	425	96.3	56.75
Oct.	17.17	1,138	32.3	614	489	96.6	59.20
Nov.	14.45	1,091	26.0	627	461	96.3	57.09
Dec.	12.00	1,035	23.5	539	408	96.4	65.44
Ave.	14.33	1,047	28.6	617	405	95.4	56.16

TABLE 15—*Continued*

Year and Month	U. S. Portland Cement Production (million bbls.)	U. S. Meat Consumption (million pounds)	U. S. Boot and Shoe Production (million pairs)	U. S. Cotton Consumption (thousand bales)	U. S. Exports (million dollars)	U. S. D. of L. Wholesale Commodity Price Index (1926 = 100)	U. S. Bank Debits (billion dollars)
1928							
Jan.	9.77	1,121	26.2	586	411	96.4	62.89
Feb.	8.80	1,068	29.6	573	371	95.8	54.49
Mar.	10.22	1,051	32.3	581	421	95.5	70.63
Apr.	13.47	947	26.6	525	364	96.6	67.00
May	17.21	1,100	26.4	577	423	97.5	71.62
June	17.50	1,057	27.2	510	389	96.7	72.49
July	17.47	986	28.1	440	379	97.4	58.98
Aug.	18.76	1,002	34.9	526	379	97.6	58.50
Sept.	17.88	1,072	31.0	492	422	98.6	63.18
Oct.	17.53	1,149	33.3	616	550	96.7	72.89
Nov.	15.07	1,084	26.4	611	545	95.8	71.35
Dec.	12.19	1,031	21.9	533	476	95.8	82.39
Ave.	14.66	1,056	28.6	548	427	96.7	67.20
1929							
Jan.	9.88	1,106	27.2	668	488	95.9	82.81
Feb.	8.52	916	27.7	595	442	95.4	70.77
Mar.	9.97	983	30.9	632	490	96.1	83.52
Apr.	13.75	1,021	29.4	632	425	95.5	74.75
May	16.15	1,087	29.2	669	385	94.7	76.54
June	16.80	1,016	28.1	569	393	95.2	69.67
July	17.32	1,072	30.2	547	403	96.5	77.63
Aug.	18.59	1,086	36.4	559	381	96.3	77.34
Sept.	17.22	1,087	34.8	546	438	96.1	77.62
Oct.	16.73	1,224	37.2	640	429	95.1	95.53
Nov.	14.05	1,089	27.7	541	442	93.5	82.09
Dec.	11.22	1,010	22.5	453	427	93.3	66.75
Ave.	14.18	1,058	30.1	587	437	95.3	77.92
1930							
Jan.	8.50	1,159	26.5	576	411	92.5	60.42
Feb.	8.16	872	25.9	494	349	91.4	52.63
Mar.	11.23	960	28.6	508	370	90.2	65.72
Apr.	13.52	1,017	29.0	532	332	90.0	62.95
May	17.25	1,076	24.5	473	320	88.8	61.81
June	17.24	1,007	23.9	405	295	86.8	62.31
July	17.18	1,025	24.1	379	267	84.4	52.74
Aug.	17.82	1,020	28.4	353	291	84.3	45.99
Sept.	16.12	1,045	29.3	393	312	84.4	48.64
Oct.	14.41	1,165	27.7	443	327	83.0	54.46
Nov.	11.10	936	18.5	415	289	81.3	42.18
Dec.	8.48	1,062	17.5	406	275	79.6	52.11
Ave.	13.14	1,029	27.4	448	320	86.4	55.16
1931							
Jan.	6.60	1,102	19.9	450	250	78.2	46.25
Feb.	5.92	895	24.0	433	224	76.8	38.03
Mar.	8.25	998	29.4	491	236	76.0	47.01
Apr.	11.25	1,004	29.9	509	215	74.8	46.44
May	14.01	1,060	28.4	465	204	73.2	43.93
June	14.12	1,045	27.8	454	187	72.1	45.30
July	13.90	1,012	28.6	451	181	72.0	39.45
Aug.	13.55	1,043	33.5	426	165	72.1	34.03
Sept.	12.09	1,067	31.3	464	180	71.2	36.70
Oct.	10.76	1,195	25.4	462	205	70.3	38.80
Nov.	8.17	1,020	18.5	429	194	70.2	29.07
Dec.	5.97	1,080	20.0	416	184	68.6	36.35
Ave.	10.38	1,043	29.4	454	202	73.0	40.11

TABLE 15—*Continued*

Year and Month	U. S. Portland Cement Production (million bbls.)	U. S. Meat Consumption (million pounds)	U. S. Boot and Shoe Production (million pairs)	U. S. Cotton Consumption (thousand bales)	U. S. Exports (million dollars)	U. S. D. of L. Wholesale Commodity Price Index (1926 = 100)	U. S. Bank Debits billion) dollars)
1932							
Jan.	5.03	1,098	21.2	435	150	67.3	33.57
Feb.	3.97	955	26.0	451	154	66.3	27.25
Mar.	4.85	1,015	30.7	489	155	66.0	29.89
Apr.	5.48	1,032	26.0	366	135	65.5	29.82
May	6.91	1,033	22.5	332	132	64.4	25.41
June	7.92	1,018	23.6	323	114	63.9	27.10
July	7.66	956	20.4	279	107	64.5	25.24
Aug.	7.84	1,002	30.8	404	109	65.2	25.22
Sept.	8.21	1,096	33.9	493	132	65.3	25.93
Oct.	7.94	1,088	33.1	502	153	64.4	25.30
Nov.	6.46	1,042	25.1	502	139	63.9	20.75
Dec.	4.25	1,014	20.0	440	132	62.6	26.79
Ave.	6.38	1,029	26.1	418	134	64.8	26.86
1933							
Jan.	2.96	1,061	22.7	470	121	61.0	24.47
Feb.	2.78	919	26.4	441	107	59.8	22.44
Mar.	3.68	993	28.6	495	108	60.2	no data
Apr.	4.18	1,030	27.6	470	105	60.4	22.63
May	6.26	1,107	33.0	621	114	62.7	25.49
June	7.80	1,095	34.9	697	120	65.0	29.71
July	8.61	1,051	33.7	601	144	68.9	31.23
Aug.	8.22	1,159	37.0	589	132	69.5	25.45
Sept.	5.64	1,163	31.2	499	160	70.8	25.56
Oct.	5.04	1,205	31.5	504	193	71.2	26.31
Nov.	4.67	1,160	23.7	475	184	71.1	24.13
Dec.	3.53	1,038	20.1	348	193	70.8	26.30
Ave.	5.28	1,082	29.2	518	140	65.9	25.70
1934							
Jan.	3.78	1,272	26.0	508	172	72.2	27.22
Feb.	4.17	1,000	30.1	477	163	73.6	25.02
Mar.	5.26	1,052	35.6	545	191	73.7	29.68
Apr.	6.54	1,015	34.4	513	179	73.3	31.23
May	8.55	1,178	34.1	519	160	73.7	28.76
June	8.81	1,084	28.5	363	171	74.6	30.14
July	8.14	971	28.1	360	162	74.8	27.75
Aug.	7.84	1,062	35.6	419	172	76.4	25.71
Sept.	7.68	959	28.2	295	191	77.6	24.01
Oct.	6.68	1,154	28.7	523	206	76.5	26.75
Nov.	5.78	1,086	23.9	480	195	76.5	24.75
Dec.	4.45	960	23.2	417	171	76.9	30.92
Ave.	6.47	1,066	29.8	452	178	74.9	27.66
1935							
Jan.	3.20	999	29.6	551	176	78.8	29.98
Feb.	3.05	776	30.9	480	163	79.5	25.66
Mar.	4.30	825	34.2	482	185	79.4	31.65
Apr.	6.14	882	34.6	468	164	80.1	31.55
May	8.22	917	31.3	470	166	80.2	30.11
June	8.76	808	27.2	384	170	79.8	31.48
July	8.02	871	32.3	391	173	79.4	33.29
Aug.	7.24	876	37.2	408	172	80.5	30.27
Sept.	7.17	834	34.0	451	199	80.7	29.03
Oct.	7.51	1,015	36.0	553	221	80.5	32.58
Nov.	7.09	914	27.7	512	270	80.6	32.23
Dec.	5.80	914	29.0	500	224	80.9	36.36
Ave.	6.73	886	32.0	471	190	80.0	31.18

TABLE 15—*Continued*

Year and Month	U. S. Portland Cement Production (million bbls.)	U. S. Meat Consumption (million pounds)	U. S. Boot and Shoe Production (million pairs)	U. S. Cotton Consumption (thousand bales)	U. S. Exports (million dollars)	U. S. D. of L. Wholesale Commodity Price Index (1926 = 100)	U. S. Bank Debits (billion dollars)
1936							
Jan.	3.65	1,008	33.4	591	199	80.6	35.43
Feb.	3.48	844	33.0	516	182	80.6	31.57
Mar.	5.31	942	34.8	551	195	79.6	37.50
Apr.	8.61	961	33.4	577	193	79.7	34.78
May	11.10	957	20.3	531	201	78.6	33.23
June	11.38	1,033	29.4	555	186	79.2	37.50
July	11.50	1,040	35.7	607	180	80.5	34.82
Aug.	12.57	1,001	40.5	575	179	81.6	31.47
Sept.	12.35	1,075	41.0	630	221	81.6	33.24
Oct.	12.47	1,195	39.9	651	265	81.5	37.31
Nov.	10.98	1,059	30.3	626	226	82.4	35.87
Dec.	8.97	1,047	33.4	695	230	84.2	45.90
Ave.	9.36	1,014	34.6	592	205	80.8	35.72
1937							
Jan.	6.62	1,008	37.1	679	223	85.9	39.49
Feb.	5.84	860	39.6	666	233	86.3	34.54
Mar.	8.44	1,040	46.1	777	257	87.8	42.01
Apr.	10.40	998	40.3	719	269	88.0	37.14
May	11.63	941	35.4	670	290	87.4	34.42
June	11.16	1,002	34.4	681	265	87.2	36.46
July	11.60	927	34.8	583	268	87.9	36.91
Aug.	11.89	938	38.6	604	277	87.5	31.90
Sept.	11.23	1,031	34.0	601	297	87.4	33.37
Oct.	11.37	1,034	29.1	524	333	85.4	36.09
Nov.	9.25	983	21.3	483	315	83.3	31.60
Dec.	7.05	1,054	21.0	432	323	81.7	39.11
Ave.	9.71	985	34.3	618	279	86.3	36.09
1938							
Jan.	4.53	1,039	25.7	433	289	80.9	32.08
Feb.	3.92	883	30.3	427	262	79.8	25.55
Mar.	5.88	989	37.5	513	275	79.7	32.12
Apr.	7.98	953	33.5	413	275	78.7	31.17
May	10.36	996	30.5	426	257	78.1	28.84
June	10.54	999	26.9	443	233	78.3	32.80
July	10.97	965	30.7	449	228	78.8	30.51
Aug.	11.01	1,017	42.2	559	231	78.1	28.27
Sept.	10.56	1,070	38.3	533	246	78.3	29.53
Oct.	11.56	1,097	35.0	544	278	77.6	33.24
Nov.	10.18	1,092	30.1	596	252	77.5	29.46
Dec.	8.07	1,040	30.0	566	269	77.0	39.97
Ave.	8.80	1,012	32.5	492	258	78.6	31.13
1939							
Jan.	5.30	1,057	34.1	598	213	76.9	32.39
Feb.	5.51	899	35.9	563	219	76.9	27.58
Mar.	8.17	1,064	42.9	650	268	76.7	34.49
Apr.	9.67	943	33.1	543	231	76.2	30.14
May	11.19	1,105	32.7	606	249	76.2	31.93
June	11.95	1,073	32.3	578	236	75.6	33.99
July	12.64	1,053	34.2	521	230	75.4	30.48
Aug.	12.37	1,138	43.9	628	251	75.0	30.61
Sept.	11.94	1,132	36.8	625	289	79.1	33.66
Oct.	12.54	1,169	37.2	687	332	79.4	32.71
Nov.	11.05	1,157	32.1	719	293	79.2	31.68
Dec.	9.49	1,156	28.7	653	368	79.2	40.02
Ave.	10.15	1,079	35.3	614	265	77.1	32.47

TABLE 15—*Continued*

Year and Month	U. S. Portland Cement Production (million bbls.)	U. S. Meat Consumption (million pounds)	U. S. Boot and Shoe Production (million pairs)	U. S. Cotton Consumption (thousand bales)	U. S. Exports (million dollars)	U. S. D. of L. Wholesale Commodity Price Index (1926 = 100)	U. S. Bank Debits (billion dollars)
1940							
Jan.	6.21	1,273	33.8	732	369	79.4	34.72
Feb.	5.04	1,054	35.6	662	347	78.7	29.48
Mar.	7.92	1,132	34.5	627	352	78.4	34.74
Apr.	10.04	1,167	31.1	623	324	78.6	34.77
May	12.63	1,200	29.4	642	325	78.4	34.20
June	12.49	1,144	27.9	565	351	77.5	31.96
July	12.29	1,152	33.6	623	317	77.7	31.85
Aug.	12.71	1,228	39.3	655	350	77.4	29.92
Sept.	13.11	1,167	34.9	639	295	78.0	30.86
Oct.	13.94	1,365	36.7	771	344	78.7	36.32
Nov.	12.73	1,289	30.4	744	328	79.6	35.77
Dec.	11.15	1,202	31.1	776	322	80.0	42.95
Ave.	10.86	1,198	33.2	672	335	78.5	33.96

Source: Survey of Current Business

TABLE 16

Economic Measures of Debt-Paying Capacity

State	Per Cent of Population Making Federal Income Tax Returns, 1936	Retail Sales per Capita, 1935	Motor Vehicle Registration per 1000 Population, 1936	Estimated Taxable Property Valuation per Capita, 1936	Savings Deposits per Capita, 1936
Alabama	.92	$118	104	$ 652	$ 29
Arizona	3.52	287	283	1,357	57
Arkansas	.72	120	107	639	21
California	6.22	385	384	2,280	342
Colorado	3.21	278	296	1,484	83
Connecticut	5.86	324	230	1,930	512
Delaware	4.51	286	230	1,593	272
Florida	2.40	261	236	906	38
Georgia	1.54	160	134	698	33
Idaho	2.31	291	274	1,090	50
Illinois	4.49	266	212	1,773	110
Indiana	2.42	226	262	1,400	79
Iowa	2.33	255	286	1,596	75
Kansas	2.17	237	306	1,392	42
Kentucky	1.38	136	129	866	51
Louisiana	1.92	161	142	676	44
Maine	2.60	273	225	969	288
Maryland	5.62	269	226	1,570	240
Massachusetts	5.94	340	185	1,710	583
Michigan	3.70	293	287	1,338	110
Minnesota	2.77	311	297	1,550	139
Mississippi	.69	88	102	452	30
Missouri	2.83	236	204	1,358	74
Montana	3.73	356	315	1,797	74
Nebraska	2.37	263	303	1,430	49
Nevada	6.49	432	385	2,460	102
New Hampshire	3.55	301	241	1,170	441
New Jersey	5.25	284	218	1,960	274
New Mexico	2.27	207	258	905	24
New York	6.72	355	190	2,370	544
North Carolina	1.06	135	146	859	27
North Dakota	1.51	214	238	1,321	42
Ohio	3.73	252	265	1,535	145
Oklahoma	1.82	171	210	743	30
Oregon	3.36	333	327	1,400	95
Pennsylvania	3.91	248	189	1,590	230
Rhode Island	4.92	319	234	1,994	493
South Carolina	.91	134	150	625	15
South Dakota	1.43	211	269	1,463	39
Tennessee	1.49	171	133	729	49
Texas	2.21	212	242	1,262	31
Utah	2.74	255	226	1,117	106
Vermont	2.91	262	221	905	380
Virginia	2.06	178	156	1,305	91
Washington	4.33	321	304	1,425	116
West Virginia	2.06	182	153	957	64
Wisconsin	3.71	299	287	1,466	133
Wyoming	3.89	354	329	1,950	88

Source: **Trule, Edna, *Resources and Debts of the Forty-Eight States, 1937*, p. 98. Dun and Bradstreet, Inc.**

Table 17

(Assets and Total Taxes Paid for the Fiscal Year 1939 by 100 Corporations having total assets under $70,000,000)
(millions of dollars)

Total Assets	Total Taxes (all kinds)	Total Assets	Total Taxes (all kinds)	Total Assets	Total Taxes (all kinds)
14.88	.75	7.05	.39	6.76	.37
44.19	1.85	11.99	.64	18.14	2.00
33.64	1.24	13.68	.89	20.76	1.00
4.79	.14	30.06	2.99	4.17	.26
33.90	1.05	56.43	.49	9.90	.14
9.21	.22	10.67	1.14	39.17	.85
55.29	2.14	53.75	2.12	18.46	1.06
22.42	.65	15.80	.56	61.92	2.87
61.07	2.41	50.98	1.33	15.87	1.26
24.25	1.35	29.65	1.58	6.65	.23
34.91	1.91	46.95	2.54	20.24	.54
49.50	2.77	55.11	2.04	31.67	.84
9.26	.94	22.04	.63	56.11	1.67
14.82	.84	34.86	.60	6.49	.42
26.49	.77	28.59	1.26	28.01	3.10
36.67	2.48	16.73	.43	52.22	1.40
34.78	1.09	22.94	1.44	23.97	1.01
42.44	.91	19.48	.47	26.58	.67
49.90	2.40	8.74	.46	59.36	2.67
66.30	2.52	49.20	.64	11.40	.55
15.84	.36	10.00	.29		
42.01	1.70	15.50	1.18		
6.15	.24	12.24	.79		
47.88	3.02	18.84	.87		
7.67	.22	13.45	.73		
43.51	1.02	6.13	.23		
52.10	1.29	47.72	3.22		
8.91	.52	47.66	1.78		
11.73	.21	18.88	.54		
34.38	1.41	12.50	.38		
24.58	1.72	10.99	1.14		
36.48	2.46	40.88	1.46		
36.99	1.48	53.41	3.55		
31.48	2.01	7.51	.35		
11.98	.45	30.35	.96		
63.34	3.09	8.23	.32		
12.43	.25	59.97	1.71		
16.60	1.93	5.71	.35		
25.22	2.94	33.48	1.20		
42.26	3.73	16.29	.43		

Source: Survey of Taxes for 1939, American Federation of Investors, Inc.

Table 18

Relationship Between Yield at Market Price of Telephone, Gas, and Electric Preferred Stocks and Number of Times Dividend and Prior Requirements Were Earned Based upon Market Prices in October, 1935

Name of Company	Yield at Market Price October, 1935	Times Requirements Earned in 1935
Alabama Power Company	8.62	1.09
Appalachian Electric Power Company	6.12	1.26
Associated Telephone Company, Ltd.	5.77	1.70
Atlanta Gas Light Company	6.42	1.73
Bangor Hydro-Electric Company, The	6.06	1.37
Bell Telephone Company of Pennsylvania	5.42	1.96
Chicago District Electric Generating Corp.	6.03	1.20
Cleveland Electric Illuminating Company	4.37	2.95
Consolidated Gas Company of New York	5.03	2.16
Consumers Power Company (Maine)	5.00	1.41
Duquesne Light Company, 1st Pfd.	4.43	3.03
Georgia Power Company	7.31	1.17
Gulf States Utilities Company	6.59	1.06
Indianapolis Power & Light Company	7.75	1.10
Jersey Central Power & Light	7.91	1.08
Kansas Gas & Electric Company	6.73	1.35
Kentucky Utilities Company	7.84	1.25
Kings County Lighting Company, Series D	6.33	1.29
Long Island Lighting Company, Series B	8.57	1.25
Louisiana Power and Light Company	6.94	1.38
Metropolitan Edison Company	5.85	1.40
Milwaukee Electric Railway & Light Company (Issue of 1921)	7.50	1.10
New York Power & Light Corporation	6.60	1.23
Ohio Edison Company	6.14	1.27
Ohio Public Service Company, 1st Pfd.	6.16	1.40
Oklahoma Gas & Electric Company	7.18	1.19
Pacific Gas & Electric Company, 1st Pfd.	5.14	1.86
Pacific Telephone & Telegraph Company	4.38	2.38
Public Service Company of Colorado, 1st Pfd.	6.19	1.55
Public Service Company of New Hampshire	5.49	1.41
Public Service Company of Northern Illinois	5.83	1.30
Public Service Electric & Gas Company	4.42	3.74
Rochester Gas & Electric Corporation, Series C	5.84	1.44
Southern California Edison Company, Ltd., Series C	5.25	1.37
Tennessee Electric Power Company, 1st Pfd.	9.09	1.12
Texas Power & Light Company	6.96	1.27
Toledo Edison Company	5.38	1.49
Virginia Electric & Power Company	5.53	1.55
Washington Water Power Company, The	6.25	2.00

Source: Federal Communications Commission: *The Problem of The "Rate of Return" in Public Utility Regulation With Special Reference to the Long Lines Department of The American Telephone and Telegraph Company,* Schedule 21, Appendix.

TABLE 19

Year	Wholesale price of raw silk (X_1)	Wholesale price of raw silk deflated (X_2)	Imports of raw silk (X_3)	Total visible supply of raw silk (X_4)	Raw silk supply in U. S. warehouses (X_5)	Index of textile manufactures (X_6)	Index of manufactures (X_7)	Raw silk deliveries to U. S. mills (X_8)	Wholesale price of rayon (X_9)
1923	$8.65	$2.80	5.2	66.3	33.4	83	86	29.9	$2.80
1924	6.25	2.11	5.1	81.8	37.5	72	81	30.6	2.11
1925	6.57	2.00	6.4	93.9	44.8	84	90	41.8	2.00
1926	6.19	1.81	6.5	94.7	36.8	84	95	41.8	1.81
1927	5.44	1.49	7.2	109.2	46.8	92	95	45.9	1.49
1928	5.07	1.50	7.4	107.5	44.7	87	100	47.6	1.50
1929	4.93	1.25	8.2	121.6	53.8	94	110	51.6	1.25
1930	3.42	1.06	6.8	199.4	50.6	74	91	48.5	1.06
1931	2.40	.76	7.5	235.0	45.4	79	74	49.6	.76
1932	1.56	.66	6.5	225.2	57.8	71	57	46.2	.66
1933	1.61	.61	6.1	231.8	62.8	88	68	39.1	.61
1934	1.30	.59	5.0	245.7	67.7	76	74	38.5	.59
1935	1.63	.57	6.0	192.0	42.7	93	87	41.4	.57
1936	1.78	.59	5.6	162.1	41.8	104	104	37.9	.59
1937	1.88	.62	5.3	149.1	44.6	106	113	35.4	.62
1938	1.71	.52	4.8	140.2	43.2	85	87	34.3	.52
1939	2.72	.52	4.6	86.9	32.2	112	108	32.0	.52
1940	2.78	.53	4.0	130.9	49.9	115	123	26.1	.53

Source: Survey of Current Business

Quoted silk price is wholesale, raw, Japanese, 13–15 denier (New York) (in dollars per pound); silk price deflated by U. S. Bureau of Labor Statistics wholesale commodity price index; imports data in millions of pounds; total visible supply and U. S. warehouse stocks of raw silk in thousands of bales; index of textile manufactures and index of manufactures by the Federal Reserve Board, 1935–1939 = 100; raw silk deliveries to U. S. mills in thousands of bales; rayon price is wholesale, 150 denier, first quality (N. Y.) in dollars per pound. All data are monthly averages for the year.

TABLE 20

Factors Affecting the Wholesale Price of Creamery Butter

Year	Wholesale price, 92 score, N. Y. (dollars per pound) (mo. av.)	Apparent consumption, (millions of pounds) (mo. av.)	Factory production (millions of pounds) (mo. av.)	Stocks, cold storage, end of month (millions of pounds) (mo. av.)
1919	$.61	68.0	72.3	67.2
1920	.61	73.2	72.0	59.8
1921	.43	89.6	87.9	52.6
1922	.41	97.6	96.1	51.0
1923	.47	105.6	103.5	47.1
1924	.43	111.2	113.0	73.7
1925	.45	114.7	113.5	61.7
1926	.44	122.7	121.0	67.7
1927	.47	124.1	124.7	71.0
1928	.47	124.2	123.9	62.0
1929	.45	129.8	133.1	81.6
1930	.37	134.4	132.9	83.2
1931	.28	142.0	139.0	55.2
1932	.21	141.5	141.2	50.3
1933	.22	139.4	146.9	91.7
1934	.26	146.6	141.2	69.2
1935	.30	138.4	136.0	71.3
1936	.33	134.8	135.8	59.6
1937	.34	137.7	135.3	64.0
1938	.28	141.7	148.8	111.2
1939	.26	152.8	146.8	111.3
1940	.30	150.7	149.6	64.4

Source: Survey of Current Business

APPENDIX B

STATISTICAL TABLES

APPENDIX TABLE I

Areas of the Normal Curve of Error in Terms of Abscissa

$\frac{x}{\sigma}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.00000	.00399	.00798	.01197	.01595	.01994	.02392	.02790	.03188	.03586
0.1	.03983	.04380	.04776	.05172	.05567	.05962	.06356	.06749	.07142	.07535
0.2	.07926	.08317	.08706	.09095	.09483	.09871	.10257	.10642	.11026	.11409
0.3	.11791	.12172	.12552	.12930	.13307	.13683	.14058	.14431	.14803	.15173
0.4	.15542	.15910	.16276	.16640	.17003	.17364	.17724	.18082	.18439	.18793
0.5	.19146	.19497	.19847	.20194	.20540	.20884	.21226	.21566	.21904	.22240
0.6	.22575	.22907	.23237	.23565	.23891	.24215	.24537	.24857	.25175	.25490
0.7	.25804	.26115	.26424	.26730	.27035	.27337	.27637	.27935	.28230	.28524
0.8	.28814	.29103	.29389	.29673	.29955	.30234	.30511	.30785	.31057	.31327
0.9	.31594	.31859	.32121	.32381	.32639	.32894	.33147	.33398	.33646	.33891
1.0	.34134	.34375	.34614	.34850	.35083	.35314	.35543	.35769	.35993	.36214
1.1	.36433	.36650	.36864	.37076	.37286	.37493	.37698	.37900	.38100	.38298
1.2	.38493	.38686	.38877	.39065	.39251	.39435	.39617	.39796	.39973	.40147
1.3	.40320	.40490	.40658	.40824	.40988	.41149	.41309	.41466	.41621	.41774
1.4	.41924	.42073	.42220	.42364	.42507	.42647	.42786	.42922	.43056	.43189
1.5	.43319	.43448	.43574	.43699	.43822	.43943	.44062	.44179	.44295	.44408
1.6	.44520	.44630	.44738	.44845	.44950	.45053	.45154	.45254	.45352	.45449
1.7	.45543	.45637	.45728	.45818	.45907	.45994	.46080	.46164	.46246	.46327
1.8	.46407	.46485	.46562	.46638	.46712	.46784	.46856	.46926	.46995	.47062
1.9	.47128	.47193	.47257	.47320	.47381	.47441	.47500	.47558	.47615	.47670
2.0	.47725	.47778	.47831	.47882	.47932	.47982	.48030	.48077	.48124	.48169
2.1	.48214	.48257	.48300	.48341	.48382	.48422	.48461	.48500	.48537	.48574
2.2	.48610	.48645	.48679	.48713	.48745	.48778	.48809	.48840	.48870	.48899
2.3	.48928	.48956	.48983	.49010	.49036	.49061	.49086	.49111	.49134	.49158
2.4	.49180	.49202	.49224	.49245	.49266	.49286	.49305	.49324	.49343	.49361
2.5	.49379	.49396	.49413	.49430	.49446	.49461	.49477	.49492	.49506	.49520
2.6	.49534	.49547	.49560	.49573	.49585	.49598	.49609	.49621	.49632	.49643
2.7	.49653	.49664	.49674	.49683	.49693	.49702	.49711	.49720	.49728	.49736
2.8	.49744	.49752	.49760	.49767	.49774	.49781	.49788	.49795	.49801	.49807
2.9	.49813	.49819	.49825	.49831	.49836	.49841	.49846	.49851	.49856	.49861
3.0	.49865	.49869	.49874	.49878	.49882	.49886	.49889	.49893	.49897	.49900
3.1	.49903	.49906	.49910	.49913	.49916	.49918	.49921	.49924	.49926	.49929
3.2	.49931	.49934	.49936	.49938	.49940	.49942	.49944	.49946	.49948	.49950
3.3	.49952	.49953	.49955	.49957	.49958	.49960	.49961	.49962	.49964	.49965
3.4	.49966	.49968	.49969	.49970	.49971	.49972	.49973	.49974	.49975	.49976

APPENDIX TABLE II[1]

Table of t

n	*P* = .05	.02	.01
1	12.706	31.821	63.657
2	4.303	6.965	9.925
3	3.182	4.541	5.841
4	2.776	3.747	4.604
5	2.571	3.365	4.032
6	2.447	3.143	3.707
7	2.365	2.998	3.499
8	2.306	2.896	3.355
9	2.262	2.821	3.250
10	2.228	2.764	3.169
11	2.201	2.718	3.106
12	2.179	2.681	3.055
13	2.160	2.650	3.012
14	2.145	2.624	2.977
15	2.131	2.602	2.947
16	2.120	2.583	2.921
17	2.110	2.567	2.898
18	2.101	2.552	2.878
19	2.093	2.539	2.861
20	2.086	2.528	2.845
21	2.080	2.518	2.831
22	2.074	2.508	2.819
23	2.069	2.500	2.807
24	2.064	2.492	2.797
25	2.060	2.485	2.787
26	2.056	2.479	2.779
27	2.052	2.473	2.771
28	2.048	2.467	2.763
29	2.045	2.462	2.756
30	2.042	2.457	2.750
∞	1.95996	2.32634	2.57582

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[1] Excerpts from Table IV, R. A. Fisher, *Statistical Methods for Research Workers.* These excerpts are printed here through the courtesy of Dr. Fisher and his publishers, Oliver and Boyd, of Edinburgh.

APPENDIX TABLE III [1]

Values of the Correlation Coefficient for Different Levels of Significance

n	*P* = .05	.02	.01
1	.996917	.9995066	.9998766
2	.95000	.98000	.990000
3	.8783	.93433	.95873
4	.8114	.8822	.91720
5	.7545	.8329	.8745
6	.7067	.7887	.8343
7	.6664	.7498	.7977
8	.6319	.7155	.7646
9	.6021	.6851	.7348
10	.5760	.6581	.7079
11	.5529	.6339	.6835
12	.5324	.6120	.6614
13	.5139	.5923	.6411
14	.4973	.5742	.6226
15	.4821	.5577	.6055
16	.4683	.5425	.5897
17	.4555	.5285	.5751
18	.4438	.5155	.5614
19	.4329	.5034	.5487
20	.4227	.4921	.5368
25	.3809	.4451	.4869
30	.3494	.4093	.4487
35	.3246	.3810	.4182
40	.3044	.3578	.3932
45	.2875	.3384	.3721
50	.2732	.3218	.3541
60	.2500	.2948	.3248
70	.2319	.2737	.3017
80	.2172	.2565	.2830
90	.2050	.2422	.2673
100	.1946	.2301	.2540

For a total correlation, *n* is 2 less than the number of pairs in the sample; for a partial correlation, the number of eliminated variates also should be subtracted.

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[1] Excerpts from Table V–A, R. A. Fisher, *Statistical Methods for Research Workers*. These excerpts are printed here through the courtesy of Dr. Fisher and his publishers, Oliver and Boyd, of Edinburgh.

APPENDIX TABLE IV

Showing the Relations between r and z for Values of z from 0 to 5[1]

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0100	.0200	.0300	.0400	.0500	.0599	.0699	.0798	.0898
.1	.0997	.1096	.1194	.1293	.1391	.1489	.1587	.1684	.1781	.1878
.2	.1974	.2070	.2165	.2260	.2355	.2449	.2543	.2636	.2729	.2821
.3	.2913	.3004	.3095	.3185	.3275	.3364	.3452	.3540	.3627	.3714
.4	.3800	.3885	.3969	.4053	.4136	.4219	.4301	.4382	.4462	.4542
.5	.4621	.4700	.4777	.4854	.4930	.5005	.5080	.5154	.5227	.5299
.6	.5370	.5441	.5511	.5581	.5649	.5717	.5784	.5850	.5915	.5980
.7	.6044	.6107	.6169	.6231	.6291	.6352	.6411	.6469	.6527	.6584
.8	.6640	.6696	.6751	.6805	.6858	.6911	.6963	.7014	.7064	.7114
.9	.7163	.7211	.7259	.7306	.7352	.7398	.7443	.7487	.7531	.7574
1.0	.7616	.7658	.7699	.7739	.7779	.7818	.7857	.7895	.7932	.7969
1.1	.8005	.8041	.8076	.8110	.8144	.8178	.8210	.8243	.8275	.8306
1.2	.8337	.8367	.8397	.8426	.8455	.8483	.8511	.8538	.8565	.8591
1.3	.8617	.8643	.8668	.8693	.8717	.8741	.8764	.8787	.8810	.8832
1.4	.8854	.8875	.8896	.8917	.8937	.8957	.8977	.8996	.9015	.9033
1.5	.9052	.9069	.9087	.9104	.9121	.9138	.9154	.9170	.9186	.9202
1.6	.9217	.9232	.9246	.9261	.9275	.9289	.9302	.9316	.9329	.9342
1.7	.9354	.9367	.9379	.9391	.9402	.9414	.9425	.9436	.9447	.9458
1.8	.9468	.9478	.9498	.9488	.9508	.9518	.9527	.9536	.9545	.9554
1.9	.9562	.9571	.9579	.9587	.9595	.9603	.9611	.9619	.9626	.9633
2.0	.9640	.9647	.9654	.9661	.9668	.9674	.9680	.9687	.9693	.9699
2.1	.9705	.9710	.9716	.9722	.9727	.9732	.9738	.9743	.9748	.9753
2.2	.9757	.9762	.9767	.9771	.9776	.9780	.9785	.9789	.9793	.9797
2.3	.9801	.9805	.9809	.9812	.9816	.9820	.9823	.9827	.9830	.9834
2.4	.9837	.9840	.9843	.9846	.9849	.9852	.9855	.9858	.9861	.9863
2.5	.9866	.9869	.9871	.9874	.9876	.9879	.9881	.9884	.9886	.9888
2.6	.9890	.9892	.9895	.9897	.9899	.9901	.9903	.9905	.9906	.9908
2.7	.9910	.9912	.9914	.9915	.9917	.9919	.9920	.9922	.9923	.9925
2.8	.9926	.9928	.9929	.9931	.9932	.9933	.9935	.9936	.9937	.9938
2.9	.9940	.9941	.9942	.9943	.9944	.9945	.9946	.9947	.9949	.9950
3.0	.9951									
4.0	.9993									
5.0	.9999									

[1] The figures in the body of the table are values of r corresponding to z-values read from the scales on the left and top of the table.

APPENDIX TABLE V[1]

Table of χ^2

n	P = .99	.95	.50	.10	.05	.02	.01
1	.000157	.00393	.455	2.706	3.841	5.412	6.635
2	.0201	.103	1.386	4.605	5.991	7.824	9.210
3	.115	.352	2.366	6.251	7.815	9.837	11.341
4	.297	.711	3.357	7.779	9.488	11.668	13.277
5	.554	1.145	4.351	9.236	11.070	13.388	15.086
6	.872	1.635	5.348	10.645	12.592	15.033	16.812
7	1.239	2.167	6.346	12.017	14.067	16.622	18.475
8	1.646	2.733	7.344	13.362	15.507	18.168	20.090
9	2.088	3.325	8.343	14.684	16.919	19.679	21.666
10	2.558	3.940	9.342	15.987	18.307	21.161	23.209
11	3.053	4.575	10.341	17.275	19.675	22.618	24.725
12	3.571	5.226	11.340	18.549	21.026	24.054	26.217
13	4.107	5.892	12.340	19.812	22.362	25.472	27.688
14	4.660	6.571	13.339	21.064	23.685	26.873	29.141
15	5.229	7.261	14.339	22.307	24.996	28.259	30.578
16	5.812	7.962	15.338	23.542	26.296	29.633	32.000
17	6.408	8.672	16.338	24.769	27.587	30.995	33.409
18	7.015	9.390	17.338	25.989	28.869	32.346	34.805
19	7.633	10.117	18.338	27.204	30.144	33.687	36.191
20	8.260	10.851	19.337	28.412	31.410	35.020	37.566
21	8.897	11.591	20.337	29.615	32.671	36.343	38.932
22	9.542	12.338	21.337	30.813	33.924	37.659	40.289
23	10.196	13.091	22.337	32.007	35.172	38.968	41.638
24	10.856	13.848	23.337	33.196	36.415	40.270	42.980
25	11.524	14.611	24.337	34.382	37.652	41.566	44.314
26	12.198	15.379	25.336	35.563	38.885	42.856	45.642
27	12.879	16.151	26.336	36.741	40.113	44.140	46.963
28	13.565	16.928	27.336	37.916	41.337	45.419	48.278
29	14.256	17.708	28.336	39.087	42.557	46.693	49.588
30	14.953	18.493	29.336	40.256	43.773	47.962	50.892

For larger values of n, the expression $\sqrt{2\chi^2} - \sqrt{2n - 1}$ may be used as a normal deviate with unit standard error.

[1] Excerpts from Table III, R. A. Fisher, *Statistical Methods for Research Workers*. These excerpts are printed here through the courtesy of Dr. Fisher and his publishers, Oliver and Boyd, of Edinburgh.

Appendix Table VI[1]

1 Per Cent Points of the Distribution of z

		Values of n_1									
		1.	2.	3.	4.	5.	6.	8.	12.	24.	∞.
Values of n_2	1	4.1535	4.2585	4.2974	4.3175	4.3297	4.3379	4.3482	4.3585	4.3689	4.3794
	2	2.2950	2.2976	2.2984	2.2988	2.2991	2.2992	2.2994	2.2997	2.2999	2.3001
	3	1.7649	1.7140	1.6915	1.6786	1.6703	1.6645	1.6569	1.6489	1.6404	1.6314
	4	1.5270	1.4452	1.4075	1.3856	1.3711	1.3609	1.3473	1.3327	1.3170	1.3000
	5	1.3943	1.2029	1.2449	1.2164	1.1974	1.1838	1.1644	1.1457	1.1239	1.0997
	6	1.3103	1.1955	1.1401	1.1068	1.0843	1.0680	1.0460	1.0218	.9948	.9643
	7	1.2526	1.1281	1.0672	1.0300	1.0048	.9864	.9614	.9335	.9020	.8658
	8	1.2106	1.0787	1.0135	.9734	.9459	.9259	.8983	.8673	.8319	.7904
	9	1.1786	1.0411	.9724	.9299	.9006	.8791	.8494	.8157	.7769	.7305
	10	1.1535	1.0114	.9399	.8954	.8646	.8419	.8104	.7744	.7324	.6816
	11	1.1333	.9874	.9136	.8674	.8354	.8116	.7785	.7405	.6958	.6408
	12	1.1166	.9677	.8919	.8443	.8111	.7864	.7520	.7122	.6640	.6061
	13	1.1027	.9511	.8737	.8248	.7907	.7652	.7295	.6882	.6386	.5761
	14	1.0909	.9370	.8581	.8082	.7732	.7471	.7103	.6675	.6159	.5500
	15	1.0807	.9249	.8448	.7939	.7582	.7314	.6937	.6496	.5961	.5269
	16	1.0719	.9144	.8331	.7814	.7450	.7177	.6791	.6339	.5786	.5064
	17	1.0641	.9051	.8229	.7705	.7335	.7057	.6663	.6199	.5630	.4879
	18	1.0572	.8970	.8138	.7607	.7232	.6950	.6549	.6075	.5516	.4712
	19	1.0511	.8897	.8057	.7521	.7140	.6854	.6447	.5964	.5366	.4560
	20	1.0457	.8831	.7985	.7443	.7058	.6768	.6355	.5864	.5253	.4421
	21	1.0408	.8772	.7920	.7372	.6984	.6690	.6272	.5773	.5150	.4294
	22	1.0363	.8719	.7860	.7309	.6916	.6620	.6196	.5691	.5056	.4176
	23	1.0322	.8670	.7806	.7251	.6855	.6555	.6127	.5615	.4969	.4068
	24	1.0285	.8626	.7757	.7197	.6799	.6496	.6064	.5545	.4890	.3967
	25	1.0251	.8585	.7712	.7148	.6747	.6442	.6006	.5481	.4816	.3872
	26	1.0220	.8548	.7670	.7103	.6699	.6392	.5952	.5422	.4748	.3784
	27	1.0191	.8513	.7631	.7062	.6655	.6346	.5902	.5367	.4685	.3701
	28	1.0164	.8481	.7595	.7023	.6614	.6303	.5856	.5316	.4626	.3624
	29	1.0139	.8451	.7562	.6987	.6576	.6263	.5813	.5269	.4570	.3550
	30	1.0116	.8423	.7531	.6954	.6540	.6226	.5773	.5224	.4519	.3481
	60	.9784	.8025	.7086	.6472	.6028	.5687	.5189	.4574	.3746	.2352
	∞	.9462	.7636	.6651	.5999	.5522	.5152	.4604	.3908	.2913	0

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[1] From Table VI, R. A. Fisher, *Statistical Methods for Research Workers.* This table is printed here through the courtesy of Dr. Fisher and his publishers, Oliver and Boyd, of Edinburgh.

APPENDIX TABLE VII[1]

5 Per Cent Points of the Distribution of z

		Values of n_1									
		1.	2.	3.	4.	5.	6.	8.	12.	24.	∞.
Values of n_2	1	2.5421	2.6479	2.6870	2.7071	2.7194	2.7276	2.7380	2.7484	2.7588	2.7693
	2	1.4592	1.4722	1.4765	1.4787	1.4800	1.4808	1.4819	1.4830	1.4840	1.4851
	3	1.1577	1.1284	1.1137	1.1051	1.0994	1.0953	1.0899	1.0842	1.0781	1.0716
	4	1.0212	.9690	.9429	.9272	.9168	.9093	.8993	.8885	.8767	.8639
	5	.9441	.8777	.8441	.8236	.8097	.7997	.7862	.7714	.7550	.7368
	6	.8948	.8188	.7798	.7558	.7394	.7274	.7112	.6931	.6729	.6499
	7	.8606	.7777	.7347	.7080	.6896	.6761	.6576	.6369	.6134	.5862
	8	.8355	.7475	.7014	.6725	.6525	.6378	.6175	.5945	.5682	.5371
	9	.8163	.7242	.6757	.6450	.6238	.6080	.5862	.5613	.5324	.4979
	10	.8012	.7058	.6553	.6232	.6009	.5843	.5611	.5346	.5035	.4657
	11	.7889	.6909	.6387	.6055	.5822	.5648	.5406	.5126	.4795	.4387
	12	.7788	.6786	.6250	.5907	.5666	.5487	.5234	.4941	.4592	.4156
	13	.7703	.6682	.6134	.5783	.5535	.5350	.5089	.4785	.4419	.3957
	14	.7630	.6594	.6036	.5677	.5423	.5233	.4964	.4649	.4269	.3782
	15	.7568	.6518	.5950	.5585	.5326	.5131	.4855	.4532	.4138	.3628
	16	.7514	.6451	.5876	.5505	.5241	.5042	.4760	.4428	.4022	.3490
	17	.7466	.6393	.5811	.5434	.5166	.4964	.4676	.4337	.3919	.3366
	18	.7424	.6341	.5753	.5371	.5099	.4894	.4602	.4255	.3827	.3253
	19	.7386	.6295	.5701	.5315	.5040	.4832	.4535	.4182	.3743	.3151
	20	.7352	.6254	.5654	.5265	.4986	.4776	.4474	.4116	.3668	.3057
	21	.7322	.6216	.5612	.5219	.4938	.4725	.4420	.4055	.3599	.2971
	22	.7294	.6182	.5574	.5178	.4894	.4679	.4370	.4001	.3536	.2892
	23	.7269	.6151	.5540	.5140	.4854	.4636	.4325	.3950	.3478	.2818
	24	.7246	.6123	.5508	.5106	.4817	.4598	.4283	.3904	.3425	.2749
	25	.7225	.6097	.5478	.5074	.4783	.4562	.4244	.3862	.3376	.2685
	26	.7205	.6073	.5451	.5045	.4752	.4529	.4209	.3823	.3330	.2625
	27	.7187	.6051	.5427	.5017	.4723	.4499	.4176	.3786	.3287	.2569
	28	.7171	.6030	.5403	.4992	.4696	.4471	.4146	.3752	.3248	.2516
	29	.7155	.6011	.5382	.4969	.4671	.4444	.4117	.3720	.3211	.2466
	30	.7141	.5994	.5362	.4947	.4648	.4420	.4090	.3691	.3176	.2419
	60	.6933	.5738	.5073	.4632	.4311	.4064	.3702	.3255	.2654	.1644
	∞	.6729	.5486	.4787	.4319	.3974	.3706	.3309	.2804	.2085	0

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[1] From Table VI, R. A. Fisher, *Statistical Methods for Research Workers.* This table is printed here through the courtesy of Dr. Fisher and his publishers, Oliver and Boyd, of Edinburgh.

TABLE VIII—5% (ROMAN TYPE) AND 1% (BOLD FACE TYPE) POINTS FOR THE DISTRIBUTION OF *F*

n_2	n_1 degrees of freedom (for greater mean square)																								n_2
	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞	
1	161 **4,052**	200 **4,999**	216 **5,403**	225 **5,625**	230 **5.764**	234 **5,859**	237 **5,928**	239 **5,981**	241 **6,022**	242 **6,056**	243 **6,082**	244 **6.106**	245 **6,142**	246 **6,169**	248 **6,208**	249 **6,234**	250 **6,258**	251 **6,286**	252 **6,302**	253 **6,323**	253 **6,334**	254 **6,352**	254 **6,361**	254 **6,366**	1
2	18.51 **98.49**	19.00 **99.01**	19.16 **99.17**	19.25 **99.25**	19.30 **99.30**	19.33 **99.33**	19.36 **99.34**	19.37 **99.36**	19.38 **99.38**	19.39 **99.40**	19.40 **99.41**	19.41 **99.42**	19.42 **99.43**	19.43 **99.44**	19.44 **99.45**	19.45 **99.46**	19.46 **99.47**	19.47 **99.48**	19.47 **99.48**	19.48 **99.49**	19.49 **99.49**	19.49 **99.49**	19.50 **99.50**	19.50 **99.50**	2
3	10.13 **34.12**	9.55 **30.81**	9.28 **29.46**	9.12 **28.71**	9.01 **28.24**	8.94 **27.91**	8.88 **27.67**	8.84 **27.49**	8.81 **27.34**	8.78 **27.23**	8.76 **27.13**	8.74 **27.05**	8.71 **26.92**	8.69 **26.83**	8.66 **26.69**	8.64 **26.60**	8.62 **26.50**	8.60 **26.41**	8.58 **26.35**	8.57 **26.27**	8.56 **26.23**	8.54 **26.18**	8.54 **26.14**	8.53 **26.12**	3
4	7.71 **21.20**	6.94 **18.00**	6.59 **16.69**	6.39 **15.98**	6.26 **15.52**	6.16 **15.21**	6.09 **14.98**	6.04 **14.80**	6.00 **14.66**	5.96 **14.54**	5.93 **14.45**	5.91 **14.37**	5.87 **14.24**	5.84 **14.15**	5.80 **14.02**	5.77 **13.93**	5.74 **13.83**	5.71 **13.74**	5.70 **13.69**	5.68 **13.61**	5.66 **13.57**	5.65 **13.52**	5.64 **13.48**	5.63 **13.46**	4
5	6.61 **16.26**	5.79 **13.27**	5.41 **12.06**	5.19 **11.39**	5.05 **10.97**	4.95 **10.67**	4.88 **10.45**	4.82 **10.27**	4.78 **10.15**	4.74 **10.05**	4.70 **9.96**	4.68 **9.89**	4.64 **9.77**	4.60 **9.68**	4.56 **9.55**	4.53 **9.47**	4.50 **9.38**	4.46 **9.29**	4.44 **9.24**	4.42 **9.17**	4.40 **9.13**	4.38 **9.07**	4.37 **9.04**	4.36 **9.02**	5
6	5.99 **13.74**	5.14 **10.92**	4.76 **9.78**	4.53 **9.15**	4.39 **8.75**	4.28 **8.47**	4.21 **8.26**	4.15 **8.10**	4.10 **7.98**	4.06 **7.87**	4.03 **7.79**	4.00 **7.72**	3.96 **7.60**	3.92 **7.52**	3.87 **7.39**	3.84 **7.31**	3.81 **7.23**	3.77 **7.14**	3.75 **7.09**	3.72 **7.02**	3.71 **6.99**	3.69 **6.94**	3.68 **6.90**	3.67 **6.88**	6
7	5.59 **12.25**	4.74 **9.55**	4.35 **8.45**	4.12 **7.85**	3.97 **7.46**	3.87 **7.19**	3.79 **7.00**	3.73 **6.84**	3.68 **6.71**	3.63 **6.62**	3.60 **6.54**	3.57 **6.47**	3.52 **6.35**	3.49 **6.27**	3.44 **6.15**	3.41 **6.07**	3.38 **5.98**	3.34 **5.90**	3.32 **5.85**	3.29 **5.78**	3.28 **5.75**	3.25 **5.70**	3.24 **5.67**	3.23 **5.65**	7
8	5.32 **11.26**	4.46 **8.65**	4.07 **7.59**	3.84 **7.01**	3.69 **6.63**	3.58 **6.37**	3.50 **6.19**	3.44 **6.03**	3.39 **5.91**	3.34 **5.82**	3.31 **5.74**	3.28 **5.67**	3.23 **5.56**	3.20 **5.48**	3.15 **5.36**	3.12 **5.28**	3.08 **5.20**	3.05 **5.11**	3.03 **5.06**	3.00 **5.00**	2.98 **4.96**	2.96 **4.91**	2.94 **4.88**	2.93 **4.86**	8
9	5.12 **10.56**	4.26 **8.02**	3.86 **6.99**	3.63 **6.42**	3.48 **6.06**	3.37 **5.80**	3.29 **5.62**	3.23 **5.47**	3.18 **5.35**	3.13 **5.26**	3.10 **5.18**	3.07 **5.11**	3.02 **5.00**	2.98 **4.92**	2.93 **4.80**	2.90 **4.73**	2.86 **4.64**	2.82 **4.56**	2.80 **4.51**	2.77 **4.45**	2.76 **4.41**	2.73 **4.36**	2.72 **4.33**	2.71 **4.31**	9
10	4.96 **10.04**	4.10 **7.56**	3.71 **6.55**	3.48 **5.99**	3.33 **5.64**	3.22 **5.39**	3.14 **5.21**	3.07 **5.06**	3.02 **4.95**	2.97 **4.85**	2.94 **4.78**	2.91 **4.71**	2.86 **4.60**	2.82 **4.52**	2.77 **4.41**	2.74 **4.33**	2.70 **4.25**	2.67 **4.17**	2.64 **4.12**	2.61 **4.05**	2.59 **4.01**	2.56 **3.96**	2.55 **3.93**	2.54 **3.91**	10
11	4.84 **9.65**	3.98 **7.20**	3.59 **6.22**	3.36 **5.67**	3.20 **5.32**	3.09 **5.07**	3.01 **4.88**	2.95 **4.74**	2.90 **4.63**	2.86 **4.54**	2.82 **4.46**	2.79 **4.40**	2.74 **4.29**	2.70 **4.21**	2.65 **4.10**	2.61 **4.02**	2.57 **3.94**	2.53 **3.86**	2.50 **3.80**	2.47 **3.74**	2.45 **3.70**	2.42 **3.66**	2.41 **3.62**	2.40 **3.60**	11
12	4.75 **9.33**	3.88 **6.93**	3.49 **5.95**	3.26 **5.41**	3.11 **5.06**	3.00 **4.82**	2.92 **4.65**	2.85 **4.50**	2.80 **4.39**	2.76 **4.30**	2.72 **4.22**	2.69 **4.16**	2.64 **4.05**	2.60 **3.98**	2.54 **3.86**	2.50 **3.78**	2.46 **3.70**	2.42 **3.61**	2.40 **3.56**	2.36 **3.49**	2.35 **3.46**	2.32 **3.41**	2.31 **3.38**	2.30 **3.36**	12
13	4.67 **9.07**	3.80 **6.70**	3.41 **5.74**	3.18 **5.20**	3.02 **4.86**	2.92 **4.62**	2.84 **4.44**	2.77 **4.30**	2.72 **4.19**	2.67 **4.10**	2.63 **4.02**	2.60 **3.96**	2.55 **3.85**	2.51 **3.78**	2.46 **3.67**	2.42 **3.59**	2.38 **3.51**	2.34 **3.42**	2.32 **3.37**	2.28 **3.30**	2.26 **3.27**	2.24 **3.21**	2.22 **3.18**	2.21 **3.16**	13

TABLE VIII—5% (ROMAN TYPE) AND 1% (BOLD FACE TYPE) POINTS FOR THE DISTRIBUTION OF F

n_2	n_1 degrees of freedom (for greater mean square)																								n_2
	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞	
14	4.60 **8.86**	3.74 **6.51**	3.34 **5.56**	3.11 **5.03**	2.96 **4.69**	2.85 **4.46**	2.77 **4.28**	2.70 **4.14**	2.65 **4.03**	2.60 **3.94**	2.56 **3.86**	2.53 **3.80**	2.48 **3.70**	2.44 **3.62**	2.39 **3.51**	2.35 **3.43**	2.31 **3.34**	2.27 **3.26**	2.24 **3.21**	2.21 **3.14**	2.19 **3.11**	2.16 **3.06**	2.14 **3.02**	2.13 **3.00**	14
15	4.54 **8.68**	3.68 **6.36**	3.29 **5.42**	3.06 **4.89**	2.90 **4.56**	2.79 **4.32**	2.70 **4.14**	2.64 **4.00**	2.59 **3.89**	2.55 **3.80**	2.51 **3.73**	2.48 **3.67**	2.43 **3.56**	2.39 **3.48**	2.33 **3.36**	2.29 **3.29**	2.25 **3.20**	2.21 **3.12**	2.18 **3.07**	2.15 **3.00**	2.12 **2.97**	2.10 **2.92**	2.08 **2.89**	2.07 **2.87**	15
16	4.49 **8.53**	3.63 **6.23**	3.24 **5.29**	3.01 **4.77**	2.85 **4.44**	2.74 **4.20**	2.66 **4.03**	2.59 **3.89**	2.54 **3.78**	2.49 **3.69**	2.45 **3.61**	2.42 **3.55**	2.37 **3.45**	2.33 **3.37**	2.28 **3.25**	2.24 **3.18**	2.20 **3.10**	2.16 **3.01**	2.13 **2.96**	2.09 **2.89**	2.07 **2.86**	2.04 **2.80**	2.02 **2.77**	2.01 **2.75**	16
17	4.45 **8.40**	3.59 **6.11**	3.20 **5.18**	2.96 **4.67**	2.81 **4.34**	2.70 **4.10**	2.62 **3.93**	2.55 **3.79**	2.50 **3.68**	2.45 **3.59**	2.41 **3.52**	2.38 **3.45**	2.33 **3.35**	2.29 **3.27**	2.23 **3.16**	2.19 **3.08**	2.15 **3.00**	2.11 **2.92**	2.08 **2.86**	2.04 **2.79**	2.02 **2.76**	1.99 **2.70**	1.97 **2.67**	1.96 **2.65**	17
18	4.41 **8.28**	3.55 **6.01**	3.16 **5.09**	2.93 **4.58**	2.77 **4.25**	2.66 **4.01**	2.58 **3.85**	2.51 **3.71**	2.46 **3.60**	2.41 **3.51**	2.37 **3.44**	2.34 **3.37**	2.29 **3.27**	2.25 **3.19**	2.19 **3.07**	2.15 **3.00**	2.11 **2.91**	2.07 **2.83**	2.04 **2.78**	2.00 **2.71**	1.98 **2.68**	1.95 **2.62**	1.93 **2.59**	1.92 **2.57**	18
19	4.38 **8.18**	3.52 **5.93**	3.13 **5.01**	2.90 **4.50**	2.74 **4.17**	2.63 **3.94**	2.55 **3.77**	2.48 **3.63**	2.43 **3.52**	2.38 **3.43**	2.34 **3.36**	2.31 **3.30**	2.26 **3.19**	2.21 **3.12**	2.15 **3.00**	2.11 **2.92**	2.07 **2.84**	2.02 **2.76**	2.00 **2.70**	1.96 **2.63**	1.94 **2.60**	1.91 **2.54**	1.90 **2.51**	1.88 **2.49**	19
20	4.35 **8.10**	3.49 **5.85**	3.10 **4.94**	2.87 **4.43**	2.71 **4.10**	2.60 **3.87**	2.52 **3.71**	2.45 **3.56**	2.40 **3.45**	2.35 **3.37**	2.31 **3.30**	2.28 **3.23**	2.23 **3.13**	2.18 **3.05**	2.12 **2.94**	2.08 **2.86**	2.04 **2.77**	1.99 **2.69**	1.96 **2.63**	1.92 **2.56**	1.90 **2.53**	1.87 **2.47**	1.85 **2.44**	1.84 **2.42**	20
21	4.32 **8.02**	3.47 **5.78**	3.07 **4.87**	2.84 **4.37**	2.68 **4.04**	2.57 **3.81**	2.49 **3.65**	2.42 **3.51**	2.37 **3.40**	2.32 **3.31**	2.28 **3.24**	2.25 **3.17**	2.20 **3.07**	2.15 **2.99**	2.09 **2.88**	2.05 **2.80**	2.00 **2.72**	1.96 **2.63**	1.93 **2.58**	1.89 **2.51**	1.87 **2.47**	1.84 **2.42**	1.82 **2.38**	1.81 **2.36**	21
22	4.30 **7.94**	3.44 **5.72**	3.05 **4.82**	2.82 **4.31**	2.66 **3.99**	2.55 **3.76**	2.47 **3.59**	2.40 **3.45**	2.35 **3.35**	2.30 **3.26**	2.26 **3.18**	2.23 **3.12**	2.18 **3.02**	2.13 **2.94**	2.07 **2.83**	2.03 **2.75**	1.98 **2.67**	1.93 **2.58**	1.91 **2.53**	1.87 **2.46**	1.84 **2.42**	1.81 **2.37**	1.80 **2.33**	1.78 **2.31**	22
23	4.28 **7.88**	3.42 **5.66**	3.03 **4.76**	2.80 **4.26**	2.64 **3.94**	2.53 **3.71**	2.45 **3.54**	2.38 **3.41**	2.32 **3.30**	2.28 **3.21**	2.24 **3.14**	2.20 **3.07**	2.14 **2.97**	2.10 **2.89**	2.04 **2.78**	2.00 **2.70**	1.96 **2.62**	1.91 **2.53**	1.88 **2.48**	1.84 **2.41**	1.82 **2.37**	1.79 **2.32**	1.77 **2.28**	1.76 **2.26**	23
24	4.26 **7.82**	3.40 **5.61**	3.01 **4.72**	2.78 **4.22**	2.62 **3.90**	2.51 **3.67**	2.43 **3.50**	2.36 **3.36**	2.30 **3.25**	2.26 **3.17**	2.22 **3.09**	2.18 **3.03**	2.13 **2.93**	2.09 **2.85**	2.02 **2.74**	1.98 **2.66**	1.94 **2.58**	1.89 **2.49**	1.86 **2.44**	1.82 **2.36**	1.80 **2.33**	1.76 **2.27**	1.74 **2.23**	1.73 **2.21**	24
25	4.24 **7.77**	3.38 **5.57**	2.99 **4.68**	2.76 **4.18**	2.60 **3.86**	2.49 **3.63**	2.41 **3.46**	2.34 **3.32**	2.28 **3.21**	2.24 **3.13**	2.20 **3.05**	2.16 **2.99**	2.11 **2.89**	2.06 **2.81**	2.00 **2.70**	1.96 **2.62**	1.92 **2.54**	1.87 **2.45**	1.84 **2.40**	1.80 **2.32**	1.77 **2.29**	1.74 **2.23**	1.72 **2.19**	1.71 **2.17**	25
26	4.22 **7.72**	3.37 **5.53**	2.98 **4.64**	2.74 **4.14**	2.59 **3.82**	2.47 **3.59**	2.39 **3.42**	2.32 **3.29**	2.27 **3.17**	2.22 **3.09**	2.18 **3.02**	2.15 **2.96**	2.10 **2.86**	2.05 **2.77**	1.99 **2.66**	1.95 **2.58**	1.90 **2.50**	1.85 **2.41**	1.82 **2.36**	1.78 **2.28**	1.76 **2.25**	1.72 **2.19**	1.70 **2.15**	1.69 **2.13**	26

The function, $F = e$ with exponent $2z$, is computed in part from Fisher's table VI (7). Additional entries are by interpolation, mostly graphical.

TABLE VIII—5% (ROMAN TYPE) AND 1% (BOLD FACE TYPE) POINTS FOR THE DISTRIBUTION OF F

n_2	n_1 degrees of freedom (for greater mean square)																								n_2
	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞	
27	4.21 **7.68**	3.35 **5.49**	2.96 **4.60**	2.73 **4.11**	2.57 **3.79**	2.46 **3.56**	2.37 **3.39**	2.30 **3.26**	2.25 **3.14**	2.20 **3.06**	2.16 **2.98**	2.13 **2.93**	2.08 **2.83**	2.03 **2.74**	1.97 **2.63**	1.93 **2.55**	1.88 **2.47**	1.84 **2.38**	1.80 **2.33**	1.76 **2.25**	1.74 **2.21**	1.71 **2.16**	1.68 **2.12**	1.67 **2.10**	27
28	4.20 **7.64**	3.34 **5.45**	2.95 **4.57**	2.71 **4.07**	2.56 **3.76**	2.44 **3.53**	2.36 **3.36**	2.29 **3.23**	2.24 **3.11**	2.19 **3.03**	2.15 **2.95**	2.12 **2.90**	2.06 **2.80**	2.02 **2.71**	1.96 **2.60**	1.91 **2.52**	1.87 **2.44**	1.81 **2.35**	1.78 **2.30**	1.75 **2.22**	1.72 **2.18**	1.69 **2.13**	1.67 **2.09**	1.65 **2.06**	28
29	4.18 **7.60**	3.33 **5.42**	2.93 **4.54**	2.70 **4.04**	2.54 **3.73**	2.43 **3.50**	2.35 **3.33**	2.28 **3.20**	2.22 **3.08**	2.18 **3.00**	2.14 **2.92**	2.10 **2.87**	2.05 **2.77**	2.00 **2.68**	1.94 **2.57**	1.90 **2.49**	1.85 **2.41**	1.80 **2.32**	1.77 **2.27**	1.73 **2.19**	1.71 **2.15**	1.68 **2.10**	1.65 **2.06**	1.64 **2.03**	29
30	4.17 **7.56**	3.32 **5.39**	2.92 **4.51**	2.69 **4.02**	2.53 **3.70**	2.42 **3.47**	2.34 **3.30**	2.27 **3.17**	2.21 **3.06**	2.16 **2.98**	2.12 **2.90**	2.09 **2.84**	2.04 **2.74**	1.99 **2.66**	1.93 **2.55**	1.89 **2.47**	1.84 **2.38**	1.79 **2.29**	1.76 **2.24**	1.72 **2.16**	1.69 **2.13**	1.66 **2.07**	1.64 **2.03**	1.62 **2.01**	30
32	4.15 **7.50**	3.30 **5.34**	2.90 **4.46**	2.67 **3.97**	2.51 **3.66**	2.40 **3.42**	2.32 **3.25**	2.25 **3.12**	2.19 **3.01**	2.14 **2.94**	2.10 **2.86**	2.07 **2.80**	2.02 **2.70**	1.97 **2.62**	1.91 **2.51**	1.86 **2.42**	1.82 **2.34**	1.76 **2.25**	1.74 **2.20**	1.69 **2.12**	1.67 **2.08**	1.64 **2.02**	1.61 **1.98**	1.59 **1.96**	32
34	4.13 **7.44**	3.28 **5.29**	2.88 **4.42**	2.65 **3.93**	2.49 **3.61**	2.38 **3.38**	2.30 **3.21**	2.23 **3.08**	2.17 **2.97**	2.12 **2.89**	2.08 **2.82**	2.05 **2.76**	2.00 **2.66**	1.95 **2.58**	1.89 **2.47**	1.84 **2.38**	1.80 **2.30**	1.74 **2.21**	1.71 **2.15**	1.67 **2.08**	1.64 **2.04**	1.61 **1.98**	1.59 **1.94**	1.57 **1.91**	34
36	4.11 **7.39**	3.26 **5.25**	2.86 **4.38**	2.63 **3.89**	2.48 **3.58**	2.36 **3.35**	2.28 **3.18**	2.21 **3.04**	2.15 **2.94**	2.10 **2.86**	2.06 **2.78**	2.03 **2.72**	1.98 **2.62**	1.93 **2.54**	1.87 **2.43**	1.82 **2.35**	1.78 **2.26**	1.72 **2.17**	1.69 **2.12**	1.65 **2.04**	1.62 **2.00**	1.59 **1.94**	1.56 **1.90**	1.55 **1.87**	36
38	4.10 **7.35**	3.25 **5.21**	2.85 **4.34**	2.62 **3.86**	2.46 **3.54**	2.35 **3.32**	2.26 **3.15**	2.19 **3.02**	2.14 **2.91**	2.09 **2.82**	2.05 **2.75**	2.02 **2.69**	1.96 **2.59**	1.92 **2.51**	1.85 **2.40**	1.80 **2.32**	1.76 **2.22**	1.71 **2.14**	1.67 **2.08**	1.63 **2.00**	1.60 **1.97**	1.57 **1.90**	1.54 **1.86**	1.53 **1.84**	38
40	4.08 **7.31**	3.23 **5.18**	2.84 **4.31**	2.61 **3.83**	2.45 **3.51**	2.34 **3.29**	2.25 **3.12**	2.18 **2.99**	2.12 **2.88**	2.07 **2.80**	2.04 **2.73**	2.00 **2.66**	1.95 **2.56**	1.90 **2.49**	1.84 **2.37**	1.79 **2.29**	1.74 **2.20**	1.69 **2.11**	1.66 **2.05**	1.61 **1.97**	1.59 **1.94**	1.55 **1.88**	1.53 **1.84**	1.51 **1.81**	40
42	4.07 **7.27**	3.22 **5.15**	2.83 **4.29**	2.59 **3.80**	2.44 **3.49**	2.32 **3.26**	2.24 **3.10**	2.17 **2.96**	2.11 **2.86**	2.06 **2.77**	2.02 **2.70**	1.99 **2.64**	1.94 **2.54**	1.89 **2.46**	1.82 **2.35**	1.78 **2.26**	1.73 **2.17**	1.68 **2.08**	1.64 **2.02**	1.60 **1.94**	1.57 **1.91**	1.54 **1.85**	1.51 **1.80**	1.49 **1.78**	42
44	4.06 **7.24**	3.21 **5.12**	2.82 **4.26**	2.58 **3.78**	2.43 **3.46**	2.31 **3.24**	2.23 **3.07**	2.16 **2.94**	2.10 **2.84**	2.05 **2.75**	2.01 **2.68**	1.98 **2.62**	1.92 **2.52**	1.88 **2.44**	1.81 **2.32**	1.76 **2.24**	1.72 **2.15**	1.66 **2.06**	1.63 **2.00**	1.58 **1.92**	1.56 **1.88**	1.52 **1.82**	1.50 **1.78**	1.48 **1.75**	44
46	4.05 **7.21**	3.20 **5.10**	2.81 **4.24**	2.57 **3.76**	2.42 **3.44**	2.30 **3.22**	2.22 **3.05**	2.14 **2.92**	2.09 **2.82**	2.04 **2.73**	2.00 **2.66**	1.97 **2.60**	1.91 **2.50**	1.87 **2.42**	1.80 **2.30**	1.75 **2.22**	1.71 **2.13**	1.65 **2.04**	1.62 **1.98**	1.57 **1.90**	1.54 **1.86**	1.51 **1.80**	1.48 **1.76**	1.46 **1.72**	46
48	4.04 **7.19**	3.19 **5.08**	2.80 **4.22**	2.56 **3.74**	2.41 **3.42**	2.30 **3.20**	2.21 **3.04**	2.14 **2.90**	2.08 **2.80**	2.03 **2.71**	1.99 **2.64**	1.96 **2.58**	1.90 **2.48**	1.86 **2.40**	1.79 **2.28**	1.74 **2.20**	1.70 **2.11**	1.64 **2.02**	1.61 **1.96**	1.56 **1.88**	1.53 **1.84**	1.50 **1.78**	1.47 **1.73**	1.45 **1.70**	48

TABLE VIII—5% (ROMAN TYPE) AND 1% (BOLD FACE TYPE) POINTS FOR THE DISTRIBUTION OF *F*

n_2	n_1 degrees of freedom (for greater mean square)																								n_2
	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞	
50	4.03 **7.17**	3.18 **5.06**	2.79 **4.20**	2.56 **3.72**	2.40 **3.41**	2.29 **3.18**	2.20 **3.02**	2.13 **2.88**	2.07 **2.78**	2.02 **2.70**	1.98 **2.62**	1.95 **2.56**	1.90 **2.46**	1.85 **2.39**	1.78 **2.26**	1.74 **2.18**	1.69 **2.10**	1.63 **2.00**	1.60 **1.94**	1.55 **1.86**	1.52 **1.82**	1.48 **1.76**	1.46 **1.71**	1.44 **1.68**	50
55	4.02 **7.12**	3.17 **5.01**	2.78 **4.16**	2.54 **3.68**	2.38 **3.37**	2.27 **3.15**	2.18 **2.98**	2.11 **2.85**	2.05 **2.75**	2.00 **2.66**	1.97 **2.59**	1.93 **2.53**	1.88 **2.43**	1.83 **2.35**	1.76 **2.23**	1.72 **2.15**	1.67 **2.06**	1.61 **1.96**	1.58 **1.90**	1.52 **1.82**	1.50 **1.78**	1.46 **1.71**	1.43 **1.66**	1.41 **1.64**	55
60	4.00 **7.08**	3.15 **4.98**	2.76 **4.13**	2.52 **3.65**	2.37 **3.34**	2.25 **3.12**	2.17 **2.95**	2.10 **2.82**	2.04 **2.72**	1.99 **2.63**	1.95 **2.56**	1.92 **2.50**	1.86 **2.40**	1.81 **2.32**	1.75 **2.20**	1.70 **2.12**	1.65 **2.03**	1.59 **1.93**	1.56 **1.87**	1.50 **1.79**	1.48 **1.74**	1.44 **1.68**	1.41 **1.63**	1.39 **1.60**	60
65	3.99 **7.04**	3.14 **4.95**	2.75 **4.10**	2.51 **3.62**	2.36 **3.31**	2.24 **3.09**	2.15 **2.93**	2.08 **2.79**	2.02 **2.70**	1.98 **2.61**	1.94 **2.54**	1.90 **2.47**	1.85 **2.37**	1.80 **2.30**	1.73 **2.18**	1.68 **2.09**	1.63 **2.00**	1.57 **1.90**	1.54 **1.84**	1.49 **1.76**	1.46 **1.71**	1.42 **1.64**	1.39 **1.60**	1.37 **1.56**	65
70	3.98 **7.01**	3.13 **4.92**	2.74 **4.08**	2.50 **3.60**	2.35 **3.29**	2.23 **3.07**	2.14 **2.91**	2.07 **2.77**	2.01 **2.67**	1.97 **2.59**	1.93 **2.51**	1.89 **2.45**	1.84 **2.35**	1.79 **2.28**	1.72 **2.15**	1.67 **2.07**	1.62 **1.98**	1.56 **1.88**	1.53 **1.82**	1.47 **1.74**	1.45 **1.69**	1.40 **1.62**	1.37 **1.56**	1.35 **1.53**	70
80	3.96 **6.96**	3.11 **4.88**	2.72 **4.04**	2.48 **3.56**	2.33 **3.25**	2.21 **3.04**	2.12 **2.87**	2.05 **2.74**	1.99 **2.64**	1.95 **2.55**	1.91 **2.48**	1.88 **2.41**	1.82 **2.32**	1.77 **2.24**	1.70 **2.11**	1.65 **2.03**	1.60 **1.94**	1.54 **1.84**	1.51 **1.78**	1.45 **1.70**	1.42 **1.65**	1.38 **1.57**	1.35 **1.52**	1.32 **1.49**	80
100	3.94 **6.90**	3.09 **4.82**	2.70 **3.98**	2.46 **3.51**	2.30 **3.20**	2.19 **2.99**	2.10 **2.82**	2.03 **2.69**	1.97 **2.59**	1.92 **2.51**	1.88 **2.43**	1.85 **2.36**	1.79 **2.26**	1.75 **2.19**	1.68 **2.06**	1.63 **1.98**	1.57 **1.89**	1.51 **1.79**	1.48 **1.73**	1.42 **1.64**	1.39 **1.59**	1.34 **1.51**	1.30 **1.46**	1.28 **1.43**	100
125	3.92 **6.84**	3.07 **4.78**	2.68 **3.94**	2.44 **3.47**	2.29 **3.17**	2.17 **2.95**	2.08 **2.79**	2.01 **2.65**	1.95 **2.56**	1.90 **2.47**	1.86 **2.40**	1.83 **2.33**	1.77 **2.23**	1.72 **2.15**	1.65 **2.03**	1.60 **1.94**	1.55 **1.85**	1.49 **1.75**	1.45 **1.68**	1.39 **1.59**	1.36 **1.54**	1.31 **1.46**	1.27 **1.40**	1.25 **1.37**	125
150	3.91 **6.81**	3.06 **4.75**	2.67 **3.91**	2.43 **3.44**	2.27 **3.14**	2.16 **2.92**	2.07 **2.76**	2.00 **2.62**	1.94 **2.53**	1.89 **2.44**	1.85 **2.37**	1.82 **2.30**	1.76 **2.20**	1.71 **2.12**	1.64 **2.00**	1.59 **1.91**	1.54 **1.83**	1.47 **1.72**	1.44 **1.66**	1.37 **1.56**	1.34 **1.51**	1.29 **1.43**	1.25 **1.37**	1.22 **1.33**	150
200	3.89 **6.76**	3.04 **4.71**	2.65 **3.88**	2.41 **3.41**	2.26 **3.11**	2.14 **2.90**	2.05 **2.73**	1.98 **2.60**	1.92 **2.50**	1.87 **2.41**	1.83 **2.34**	1.80 **2.28**	1.74 **2.17**	1.69 **2.09**	1.62 **1.97**	1.57 **1.88**	1.52 **1.79**	1.45 **1.69**	1.42 **1.62**	1.35 **1.53**	1.32 **1.48**	1.26 **1.39**	1.22 **1.33**	1.19 **1.28**	200
400	3.86 **6.70**	3.02 **4.66**	2.62 **3.83**	2.39 **3.36**	2.23 **3.06**	2.12 **2.85**	2.03 **2.69**	1.96 **2.55**	1.90 **2.46**	1.85 **2.37**	1.81 **2.29**	1.78 **2.23**	1.72 **2.12**	1.67 **2.04**	1.60 **1.92**	1.54 **1.84**	1.49 **1.74**	1.42 **1.64**	1.38 **1.57**	1.32 **1.47**	1.28 **1.42**	1.22 **1.32**	1.16 **1.24**	1.13 **1.19**	400
1000	3.85 **6.66**	3.00 **4.62**	2.61 **3.80**	2.38 **3.34**	2.22 **3.04**	2.10 **2.82**	2.02 **2.66**	1.95 **2.53**	1.89 **2.43**	1.84 **2.34**	1.80 **2.26**	1.76 **2.20**	1.70 **2.09**	1.65 **2.01**	1.58 **1.89**	1.53 **1.81**	1.47 **1.71**	1.41 **1.61**	1.36 **1.54**	1.30 **1.44**	1.26 **1.38**	1.19 **1.28**	1.13 **1.19**	1.08 **1.11**	1000
∞	3.84 **6.64**	2.99 **4.60**	2.60 **3.78**	2.37 **3.32**	2.21 **3.02**	2.09 **2.80**	2.01 **2.64**	1.94 **2.51**	1.88 **2.41**	1.83 **2.32**	1.79 **2.24**	1.75 **2.18**	1.69 **2.07**	1.64 **1.99**	1.57 **1.87**	1.52 **1.79**	1.46 **1.69**	1.40 **1.59**	1.35 **1.52**	1.28 **1.41**	1.24 **1.36**	1.17 **1.25**	1.11 **1.15**	1.00 **1.00**	∞

APPENDIX TABLE IX

Sums of the First Three Powers of the Natural Numbers from 1 to 50

n	$\Sigma(n)$	$\Sigma(n^2)$	$\Sigma(n^3)$	n	$\Sigma(n)$	$\Sigma(n^2)$	$\Sigma(n^3)$
1	1	1	1	26	351	6 201	123 201
2	3	5	9	27	378	6 930	142 884
3	6	14	36	28	406	7 714	164 836
4	10	30	100	29	435	8 555	189 225
5	15	55	225	30	465	9 455	216 225
6	21	91	441	31	496	10 416	246 016
7	28	140	784	32	528	11 440	278 784
8	36	204	1 296	33	561	12 529	314 721
9	45	285	2 025	34	595	13 685	354 025
10	55	385	3 025	35	630	14 910	396 900
11	66	506	4 356	36	666	16 206	443 556
12	78	650	6 084	37	703	17 575	494 209
13	91	819	8 281	38	741	19 019	549 081
14	105	1 015	11 025	39	780	20 540	608 400
15	120	1 240	14 400	40	820	22 140	672 400
16	136	1 496	18 496	41	861	23 821	741 321
17	153	1 785	23 409	42	903	25 585	815 409
18	171	2 109	29 241	43	946	27 434	894 916
19	190	2 470	36 100	44	990	29 370	980 100
20	210	2 870	44 100	45	1 035	31 395	1 071 225
21	231	3 311	53 361	46	1 081	33 511	1 168 561
22	253	3 795	64 009	47	1 128	35 720	1 272 384
23	276	4 324	76 176	48	1 176	38 024	1 382 976
24	300	4 900	90 000	49	1 225	40 425	1 500 625
25	325	5 525	105 625	50	1 275	42 925	1 625 625

TABLE X

COMMON LOGARITHMS OF NUMBERS

FROM 1 TO 10000

TO FIVE DECIMAL PLACES

1–100

N	Log	N	Log	N	Log	N	Log	N	Log
0	—	**20**	1.30 103	**40**	1.60 206	**60**	1.77 815	**80**	1.90 309
1	0.00 000	21	1.32 222	41	1.61 278	61	1.78 533	81	1.90 849
2	0.30 103	22	1.34 242	42	1.62 325	62	1.79 239	82	1.91 381
3	0.47 712	23	1.36 173	43	1.63 347	63	1.79 934	83	1.91 908
4	0.60 206	24	1.38 021	44	1.64 345	64	1.80 618	84	1.92 428
5	0.69 897	25	1.39 794	45	1.65 321	65	1.81 291	85	1.92 942
6	0.77 815	26	1.41 497	46	1.66 276	66	1.81 954	86	1.93 450
7	0.84 510	27	1.43 136	47	1.67 210	67	1.82 607	87	1.93 952
8	0.90 309	28	1.44 716	48	1.68 124	68	1.83 251	88	1.94 448
9	0.95 424	29	1.46 240	49	1.69 020	69	1.83 885	89	1.94 939
10	1.00 000	**30**	1.47 712	**50**	1.69 897	**70**	1.84 510	**90**	1.95 424
11	1.04 139	31	1.49 136	51	1.70 757	71	1.85 126	91	1.95 904
12	1.07 918	32	1.50 515	52	1.71 600	72	1.85 733	92	1.96 379
13	1.11 394	33	1.51 851	53	1.72 428	73	1.86 332	93	1.96 848
14	1.14 613	34	1.53 148	54	1.73 239	74	1.86 923	94	1.97 313
15	1.17 609	35	1.54 407	55	1.74 036	75	1.87 506	95	1.97 772
16	1.20 412	36	1.55 630	56	1.74 819	76	1.88 081	96	1.98 227
17	1.23 045	37	1.56 820	57	1.75 587	77	1.88 649	97	1.98 677
18	1.25 527	38	1.57 978	58	1.76 343	78	1.89 209	98	1.99 123
19	1.27 875	39	1.59 106	59	1.77 085	79	1.89 763	99	1.99 564
20	1.30 103	**40**	1.60 206	**60**	1.77 815	**80**	1.90 309	**100**	2.00 000

N	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
101	00 432	475	518	561	604	647	689	732	775	817
102	00 860	903	945	988	*030	*072	*115	*157	*199	*242
103	01 284	326	368	410	452	494	536	578	620	662
104	01 703	745	787	828	870	912	953	995	*036	*078
105	02 119	160	202	243	284	325	366	407	449	490
106	02 531	572	612	653	694	735	776	816	857	898
107	02 938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03 342	383	423	463	503	543	583	623	663	703
109	03 743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
111	04 532	571	610	650	689	727	766	805	844	883
112	04 922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05 308	346	385	423	461	500	538	576	614	652
114	05 690	729	767	805	843	881	918	956	994	*032
115	06 070	108	145	183	221	258	296	333	371	408
116	06 446	483	521	558	595	633	670	707	744	781
117	06 819	856	893	930	967	*004	*041	*078	*115	*151
118	07 188	225	262	298	335	372	408	445	482	518
119	07 555	591	628	664	700	737	773	809	846	882
120	07 918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08 279	314	350	386	422	458	493	529	565	600
122	08 636	672	707	743	778	814	849	884	920	955
123	08 991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	587	621	656
125	09 691	726	760	795	830	864	899	934	968	*003
126	10 037	072	106	140	175	209	243	278	312	346
127	10 380	415	449	483	517	551	585	619	653	687
128	10 721	755	789	823	857	890	924	958	992	*025
129	11 059	093	126	160	193	227	261	294	327	361
130	11 394	428	461	494	528	561	594	628	661	694
131	11 727	760	793	826	860	893	926	959	992	*024
132	12 057	090	123	156	189	222	254	287	320	352
133	12 385	418	450	483	516	548	581	613	646	678
134	12 710	743	775	808	840	872	905	937	969	*001
135	13 033	066	098	130	162	194	226	258	290	322
136	13 354	386	418	450	481	513	545	577	609	640
137	13 672	704	735	767	799	830	862	893	925	956
138	13 988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14 301	333	364	395	426	457	489	520	551	582
140	14 613	644	675	706	737	768	799	829	860	891
141	14 922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	15 534	564	594	625	655	685	715	746	776	806
144	15 836	866	897	927	957	987	*017	*047	*077	*107
145	16 137	167	197	227	256	286	316	346	376	406
146	16 435	465	495	524	554	584	613	643	673	702
147	16 732	761	791	820	850	879	909	938	967	997
148	17 026	056	085	114	143	173	202	231	260	289
149	17 319	348	377	406	435	464	493	522	551	580
150	17 609	638	667	696	725	754	782	811	840	869
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Parts

	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	34	33
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

Prop. Parts

N	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
151	17 898	926	955	984	*013	*041	*070	*099	*127	*156
152	18 184	213	241	270	298	327	355	384	412	441
153	18 469	498	526	554	583	611	639	667	696	724
154	18 752	780	808	837	865	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	257	285
156	19 312	340	368	396	424	451	479	507	535	562
157	19 590	618	645	673	700	728	756	783	811	838
158	19 866	893	921	948	976	*003	*030	*058	*085	*112
159	20 140	167	194	222	249	276	303	330	358	385
160	20 412	439	466	493	520	548	575	602	629	656
161	20 683	710	737	763	790	817	844	871	898	925
162	20 952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21 219	245	272	299	325	352	378	405	431	458
164	21 484	511	537	564	590	617	643	669	696	722
165	21 748	775	801	827	854	880	906	932	958	985
166	22 011	037	063	089	115	141	167	194	220	246
167	22 272	298	324	350	376	401	427	453	479	505
168	22 531	557	583	608	634	660	686	712	737	763
169	22 789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
171	23 300	325	350	376	401	426	452	477	502	528
172	23 553	578	603	629	654	679	704	729	754	779
173	23 805	830	855	880	905	930	955	980	*005	*030
174	24 055	080	105	130	155	180	204	229	254	279
175	24 304	329	353	378	403	428	452	477	502	527
176	24 551	576	601	625	650	674	699	724	748	773
177	24 797	822	846	871	895	920	944	969	993	*018
178	25 042	066	091	115	139	164	188	212	237	261
179	25 285	310	334	358	382	406	431	455	479	503
180	25 527	551	575	600	624	648	672	696	720	744
181	25 768	792	816	840	864	888	912	935	959	983
182	26 007	031	055	079	102	126	150	174	198	221
183	26 245	269	293	316	340	364	387	411	435	458
184	26 482	505	529	553	576	600	623	647	670	694
185	26 717	741	764	788	811	834	858	881	905	928
186	26 951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27 184	207	231	254	277	300	323	346	370	393
188	27 416	439	462	485	508	531	554	577	600	623
189	27 646	669	692	715	738	761	784	807	830	852
190	27 875	898	921	944	967	989	*012	*035	*058	*081
191	28 103	126	149	171	194	217	240	262	285	307
192	28 330	353	375	398	421	443	466	488	511	533
193	28 556	578	601	623	646	668	691	713	735	758
194	28 780	803	825	847	870	892	914	937	959	981
195	29 003	026	048	070	092	115	137	159	181	203
196	29 226	248	270	292	314	336	358	380	403	425
197	29 447	469	491	513	535	557	579	601	623	645
198	29 667	688	710	732	754	776	798	820	842	863
199	29 885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N	0	1	2	3	4	5	6	7	8	9

Prop. Parts

	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25
1	2.5
2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24	23
1	2.4	2.3
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

Prop. Parts

N	0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
201	30 320	341	363	384	406	428	449	471	492	514
202	30 535	557	578	600	621	643	664	685	707	728
203	30 750	771	792	814	835	856	878	899	920	942
204	30 963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31 175	197	218	239	260	281	302	323	345	366
206	31 387	408	429	450	471	492	513	534	555	576
207	31 597	618	639	660	681	702	723	744	765	785
208	31 806	827	848	869	890	911	931	952	973	994
209	32 015	035	056	077	098	118	139	160	181	201
210	32 222	243	263	284	305	325	346	366	387	408
211	32 428	449	469	490	510	531	552	572	593	613
212	32 634	654	675	695	715	736	756	777	797	818
213	32 838	858	879	899	919	940	960	980	*001	*021
214	33 041	062	082	102	122	143	163	183	203	224
215	33 244	264	284	304	325	345	365	385	405	425
216	33 445	465	486	506	526	546	566	586	606	626
217	33 646	666	686	706	726	746	766	786	806	826
218	33 846	866	885	905	925	945	965	985	*005	*025
219	34 044	064	084	104	124	143	163	183	203	223
220	34 242	262	282	301	321	341	361	380	400	420
221	34 439	459	479	498	518	537	557	577	596	616
222	34 635	655	674	694	713	733	753	772	792	811
223	34 830	850	869	889	908	928	947	967	986	*005
224	35 025	044	064	083	102	122	141	160	180	199
225	35 218	238	257	276	295	315	334	353	372	392
226	35 411	430	449	468	488	507	526	545	564	583
227	35 603	622	641	660	679	698	717	736	755	774
228	35 793	813	832	851	870	889	908	927	946	965
229	35 984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
231	36 361	380	399	418	436	455	474	493	511	530
232	36 549	568	586	605	624	642	661	680	698	717
233	36 736	754	773	791	810	829	847	866	884	903
234	36 922	940	959	977	996	*014	*033	*051	*070	*088
235	37 107	125	144	162	181	199	218	236	254	273
236	37 291	310	328	346	365	383	401	420	438	457
237	37 475	493	511	530	548	566	585	603	621	639
238	37 658	676	694	712	731	749	767	785	803	822
239	37 840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
241	38 202	220	238	256	274	292	310	328	346	364
242	38 382	399	417	435	453	471	489	507	525	543
243	38 561	578	596	614	632	650	668	686	703	721
244	38 739	757	775	792	810	828	846	863	881	899
245	38 917	934	952	970	987	*005	*023	*041	*058	*076
246	39 094	111	129	146	164	182	199	217	235	252
247	39 270	287	305	322	340	358	375	393	410	428
248	39 445	463	480	498	515	533	550	568	585	602
249	39 620	637	655	672	690	707	724	742	759	777
250	39 794	811	829	846	863	881	898	915	933	950
N	0	1	2	3	4	5	6	7	8	9

Prop. Parts

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

	19
1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

Prop. Parts

X. 250—LOGARITHMS OF NUMBERS—300

Prop. Parts	18	17	16	15	14
1	1.8	1.7	1.6	1.5	1.4
2	3.6	3.4	3.2	3.0	2.8
3	5.4	5.1	4.8	4.5	4.2
4	7.2	6.8	6.4	6.0	5.6
5	9.0	8.5	8.0	7.5	7.0
6	10.8	10.2	9.6	9.0	8.4
7	12.6	11.9	11.2	10.5	9.8
8	14.4	13.6	12.8	12.0	11.2
9	16.2	15.3	14.4	13.5	12.6

N	0	1	2	3	4	5	6	7	8	9
250	39 794	811	829	846	863	881	898	915	933	950
251	39 967	985	*002	*019	*037	*054	*071	*088	*106	*123
252	40 140	157	175	192	209	226	243	261	278	295
253	40 312	329	346	364	381	398	415	432	449	466
254	40 483	500	518	535	552	569	586	603	620	637
255	40 654	671	688	705	722	739	756	773	790	807
256	40 824	841	858	875	892	909	926	943	960	976
257	40 993	*010	*027	*044	*061	*078	*095	*111	*128	*145
258	41 162	179	196	212	229	246	263	280	296	313
259	41 330	347	363	380	397	414	430	447	464	481
260	41 497	514	531	547	564	581	597	614	631	647
261	41 664	681	697	714	731	747	764	780	797	814
262	41 830	847	863	880	896	913	929	946	963	979
263	41 996	*012	*029	*045	*062	*078	*095	*111	*127	*144
264	42 160	177	193	210	226	243	259	275	292	308
265	42 325	341	357	374	390	406	423	439	455	472
266	42 488	504	521	537	553	570	586	602	619	635
267	42 651	667	684	700	716	732	749	765	781	797
268	42 813	830	846	862	878	894	911	927	943	959
269	42 975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43 136	152	169	185	201	217	233	249	265	281
271	43 297	313	329	345	361	377	393	409	425	441
272	43 457	473	489	505	521	537	553	569	584	600
273	43 616	632	648	664	680	696	712	727	743	759
274	43 775	791	807	823	838	854	870	886	902	917
275	43 933	949	965	981	996	*012	*028	*044	*059	*075
276	44 091	107	122	138	154	170	185	201	217	232
277	44 248	264	279	295	311	326	342	358	373	389
278	44 404	420	436	451	467	483	498	514	529	545
279	44 560	576	592	607	623	638	654	669	685	700
280	44 716	731	747	762	778	793	809	824	840	855
281	44 871	886	902	917	932	948	963	979	994	*010
282	45 025	040	056	071	086	102	117	133	148	163
283	45 179	194	209	225	240	255	271	286	301	317
284	45 332	347	362	378	393	408	423	439	454	469
285	45 484	500	515	530	545	561	576	591	606	621
286	45 637	652	667	682	697	712	728	743	758	773
287	45 788	803	818	834	849	864	879	894	909	924
288	45 939	954	969	984	*000	*015	*030	*045	*060	*075
289	46 090	105	120	135	150	165	180	195	210	225
290	46 240	255	270	285	300	315	330	345	359	374
291	46 389	404	419	434	449	464	479	494	509	523
292	46 538	553	568	583	598	613	627	642	657	672
293	46 687	702	716	731	746	761	776	790	805	820
294	46 835	850	864	879	894	909	923	938	953	967
295	46 982	997	*012	*026	*041	*056	*070	*085	*100	*114
296	47 129	144	159	173	188	202	217	232	246	261
297	47 276	290	305	319	334	349	363	378	392	407
298	47 422	436	451	465	480	494	509	524	538	553
299	47 567	582	596	611	625	640	654	669	683	698
300	47 712	727	741	756	770	784	799	813	828	842
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N	0	1	2	3	4	5	6	7	8	9
300	47 712	727	741	756	770	784	799	813	828	842
301	47 857	871	885	900	914	929	943	958	972	986
302	48 001	015	029	044	058	073	087	101	116	130
303	48 144	159	173	187	202	216	230	244	259	273
304	48 287	302	316	330	344	359	373	387	401	416
305	48 430	444	458	473	487	501	515	530	544	558
306	48 572	586	601	615	629	643	657	671	686	700
307	48 714	728	742	756	770	785	799	813	827	841
308	48 855	869	883	897	911	926	940	954	968	982
309	48 996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
311	49 276	290	304	318	332	346	360	374	388	402
312	49 415	429	443	457	471	485	499	513	527	541
313	49 554	568	582	596	610	624	638	651	665	679
314	49 693	707	721	734	748	762	776	790	803	817
315	49 831	845	859	872	886	900	914	927	941	955
316	49 969	982	996	*010	*024	*037	*051	*065	*079	*092
317	50 106	120	133	147	161	174	188	202	215	229
318	50 243	256	270	284	297	311	325	338	352	365
319	50 379	393	406	420	433	447	461	474	488	501
320	50 515	529	542	556	569	583	596	610	623	637
321	50 651	664	678	691	705	718	732	745	759	772
322	50 786	799	813	826	840	853	866	880	893	907
323	50 920	934	947	961	974	987	*001	*014	*028	*041
324	51 055	068	081	095	108	121	135	148	162	175
325	51 188	202	215	228	242	255	268	282	295	308
326	51 322	335	348	362	375	388	402	415	428	441
327	51 455	468	481	495	508	521	534	548	561	574
328	51 587	601	614	627	640	654	667	680	693	706
329	51 720	733	746	759	772	786	799	812	825	838
330	51 851	865	878	891	904	917	930	943	957	970
331	51 983	996	*009	*022	*035	*048	*061	*075	*088	*101
332	52 114	127	140	153	166	179	192	205	218	231
333	52 244	257	270	284	297	310	323	336	349	362
334	52 375	388	401	414	427	440	453	466	479	492
335	52 504	517	530	543	556	569	582	595	608	621
336	52 634	647	660	673	686	699	711	724	737	750
337	52 763	776	789	802	815	827	840	853	866	879
338	52 892	905	917	930	943	956	969	982	994	*007
339	53 020	033	046	058	071	084	097	110	122	135
340	53 148	161	173	186	199	212	224	237	250	263
341	53 275	288	301	314	326	339	352	364	377	390
342	53 403	415	428	441	453	466	479	491	504	517
343	53 529	542	555	567	580	593	605	618	631	643
344	53 656	668	681	694	706	719	732	744	757	769
345	53 782	794	807	820	832	845	857	870	882	895
346	53 908	920	933	945	958	970	983	995	*008	*020
347	54 033	045	058	070	083	095	108	120	133	145
348	54 158	170	183	195	208	220	233	245	258	270
349	54 283	295	307	320	332	345	357	370	382	394
350	54 407	419	432	444	456	469	481	494	506	518
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Parts

	15
1	1.5
2	3.0
3	4.5
4	6.0
5	7.5
6	9.0
7	10.5
8	12.0
9	13.5

	14
1	1.4
2	2.8
3	4.2
4	5.6
5	7.0
6	8.4
7	9.8
8	11.2
9	12.6

	13
1	1.3
2	2.6
3	3.9
4	5.2
5	6.5
6	7.8
7	9.1
8	10.4
9	11.7

	12
1	1.2
2	2.4
3	3.6
4	4.8
5	6.0
6	7.2
7	8.4
8	9.6
9	10.8

Prop. Parts

N	0	1	2	3	4	5	6	7	8	9
350	54 407	419	432	444	456	469	481	494	506	518
351	54 531	543	555	568	580	593	605	617	630	642
352	54 654	667	679	691	704	716	728	741	753	765
353	54 777	790	802	814	827	839	851	864	876	888
354	54 900	913	925	937	949	962	974	986	998	*011
355	55 023	035	047	060	072	084	096	108	121	133
356	55 145	157	169	182	194	206	218	230	242	255
357	55 267	279	291	303	315	328	340	352	364	376
358	55 388	400	413	425	437	449	461	473	485	497
359	55 509	522	534	546	558	570	582	594	606	618
360	55 630	642	654	666	678	691	703	715	727	739
361	55 751	763	775	787	799	811	823	835	847	859
362	55 871	883	895	907	919	931	943	955	967	979
363	55 991	*003	*015	*027	*038	*050	*062	*074	*086	*098
364	56 110	122	134	146	158	170	182	194	205	217
365	56 229	241	253	265	277	289	301	312	324	336
366	56 348	360	372	384	396	407	419	431	443	455
367	56 467	478	490	502	514	526	538	549	561	573
368	56 585	597	608	620	632	644	656	667	679	691
369	56 703	714	726	738	750	761	773	785	797	808
370	56 820	832	844	855	867	879	891	902	914	926
371	56 937	949	961	972	984	996	*008	*019	*031	*043
372	57 054	066	078	089	101	113	124	136	148	159
373	57 171	183	194	206	217	229	241	252	264	276
374	57 287	299	310	322	334	345	357	368	380	392
375	57 403	415	426	438	449	461	473	484	496	507
376	57 519	530	542	553	565	576	588	600	611	623
377	57 634	646	657	669	680	692	703	715	726	738
378	57 749	761	772	784	795	807	818	830	841	852
379	57 864	875	887	898	910	921	933	944	955	967
380	57 978	990	*001	*013	*024	*035	*047	*058	*070	*081
381	58 092	104	115	127	138	149	161	172	184	195
382	58 206	218	229	240	252	263	274	286	297	309
383	58 320	331	343	354	365	377	388	399	410	422
384	58 433	444	456	467	478	490	501	512	524	535
385	58 546	557	569	580	591	602	614	625	636	647
386	58 659	670	681	692	704	715	726	737	749	760
387	58 771	782	794	805	816	827	838	850	861	872
388	58 883	894	906	917	928	939	950	961	973	984
389	58 995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59 106	118	129	140	151	162	173	184	195	207
391	59 218	229	240	251	262	273	284	295	306	318
392	59 329	340	351	362	373	384	395	406	417	428
393	59 439	450	461	472	483	494	506	517	528	539
394	59 550	561	572	583	594	605	616	627	638	649
395	59 660	671	682	693	704	715	726	737	748	759
396	59 770	780	791	802	813	824	835	846	857	868
397	59 879	890	901	912	923	934	945	956	966	977
398	59 988	999	*010	*021	*032	*043	*054	*065	*076	*086
399	60 097	108	119	130	141	152	163	173	184	195
400	60 206	217	228	239	249	260	271	282	293	304
N	0	1	2	3	4	5	6	7	8	9

Prop. Parts

	13
1	1.3
2	2.6
3	3.9
4	5.2
5	6.5
6	7.8
7	9.1
8	10.4
9	11.7

	12
1	1.2
2	2.4
3	3.6
4	4.8
5	6.0
6	7.2
7	8.4
8	9.6
9	10.8

	11
1	1.1
2	2.2
3	3.3
4	4.4
5	5.5
6	6.6
7	7.7
8	8.8
9	9.9

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

Prop. Parts

N	0	1	2	3	4	5	6	7	8	9
400	60 206	217	228	239	249	260	271	282	293	304
401	60 314	325	336	347	358	369	379	390	401	412
402	60 423	433	444	455	466	477	487	498	509	520
403	60 531	541	552	563	574	584	595	606	617	627
404	60 638	649	660	670	681	692	703	713	724	735
405	60 746	756	767	778	788	799	810	821	831	842
406	60 853	863	874	885	895	906	917	927	938	949
407	60 959	970	981	991	*002	*013	*023	*034	*045	*055
408	61 066	077	087	098	109	119	130	140	151	162
409	61 172	183	194	204	215	225	236	247	257	268
410	61 278	289	300	310	321	331	342	352	363	374
411	61 384	395	405	416	426	437	448	458	469	479
412	61 490	500	511	521	532	542	553	563	574	584
413	61 595	606	616	627	637	648	658	669	679	690
414	61 700	711	721	731	742	752	763	773	784	794
415	61 805	815	826	836	847	857	868	878	888	899
416	61 909	920	930	941	951	962	972	982	993	*003
417	62 014	024	034	045	055	066	076	086	097	107
418	62 118	128	138	149	159	170	180	190	201	211
419	62 221	232	242	252	263	273	284	294	304	315
420	62 325	335	346	356	366	377	387	397	408	418
421	62 428	439	449	459	469	480	490	500	511	521
422	62 531	542	552	562	572	583	593	603	613	624
423	62 634	644	655	665	675	685	696	706	716	726
424	62 737	747	757	767	778	788	798	808	818	829
425	62 839	849	859	870	880	890	900	910	921	931
426	62 941	951	961	972	982	992	*002	*012	*022	*033
427	63 043	053	063	073	083	094	104	114	124	134
428	63 144	155	165	175	185	195	205	215	225	236
429	63 246	256	266	276	286	296	306	317	327	337
430	63 347	357	367	377	387	397	407	417	428	438
431	63 448	458	468	478	488	498	508	518	528	538
432	63 548	558	568	579	589	599	609	619	629	639
433	63 649	659	669	679	689	699	709	719	729	739
434	63 749	759	769	779	789	799	809	819	829	839
435	63 849	859	869	879	889	899	909	919	929	939
436	63 949	959	969	979	988	998	*008	*018	*028	*038
437	64 048	058	068	078	088	098	108	118	128	137
438	64 147	157	167	177	187	197	207	217	227	237
439	64 246	256	266	276	286	296	306	316	326	335
440	64 345	355	365	375	385	395	404	414	424	434
441	64 444	454	464	473	483	493	503	513	523	532
442	64 542	552	562	572	582	591	601	611	621	631
443	64 640	650	660	670	680	689	699	709	719	729
444	64 738	748	758	768	777	787	797	807	816	826
445	64 836	846	856	865	875	885	895	904	914	924
446	64 933	943	953	963	972	982	992	*002	*011	*021
447	65 031	040	050	060	070	079	089	099	108	118
448	65 128	137	147	157	167	176	186	196	205	215
449	65 225	234	244	254	263	273	283	292	302	312
450	65 321	331	341	350	360	369	379	389	398	408
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Parts

	11
1	1.1
2	2.2
3	3.3
4	4.4
5	5.5
6	6.6
7	7.7
8	8.8
9	9.9

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

Prop. Parts

Prop. Parts	N	0	1	2	3	4	5	6	7	8	9
	450	65 321	331	341	350	360	369	379	389	398	408
	451	65 418	427	437	447	456	466	475	485	495	504
	452	65 514	523	533	543	552	562	571	581	591	600
	453	65 610	619	629	639	648	658	667	677	686	696
	454	65 706	715	725	734	744	753	763	772	782	792
	455	65 801	811	820	830	839	849	858	868	877	887
	456	65 896	906	916	925	935	944	954	963	973	982
10 1 1.0 2 2.0 3 3.0 4 4.0 5 5.0 6 6.0 7 7.0 8 8.0 9 9.0	457	65 992	*001	*011	*020	*030	*039	*049	*058	*068	*077
	458	66 087	096	106	115	124	134	143	153	162	172
	459	66 181	191	200	210	219	229	238	247	257	266
	460	66 276	285	295	304	314	323	332	342	351	361
	461	66 370	380	389	398	408	417	427	436	445	455
	462	66 464	474	483	492	502	511	521	530	539	549
	463	66 558	567	577	586	596	605	614	624	633	642
	464	66 652	661	671	680	689	699	708	717	727	736
	465	66 745	755	764	773	783	792	801	811	820	829
	466	66 839	848	857	867	876	885	894	904	913	922
	467	66 932	941	950	960	969	978	987	997	*006	*015
	468	67 025	034	043	052	062	071	080	089	099	108
	469	67 117	127	136	145	154	164	173	182	191	201
	470	67 210	219	228	237	247	256	265	274	284	293
	471	67 302	311	321	330	339	348	357	367	376	385
9 1 0.9 2 1.8 3 2.7 4 3.6 5 4.5 6 5.4 7 6.3 8 7.2 9 8.1	472	67 394	403	413	422	431	440	449	459	468	477
	473	67 486	495	504	514	523	532	541	550	560	569
	474	67 578	587	596	605	614	624	633	642	651	660
	475	67 669	679	688	697	706	715	724	733	742	752
	476	67 761	770	779	788	797	806	815	825	834	843
	477	67 852	861	870	879	888	897	906	916	925	934
	478	67 943	952	961	970	979	988	997	*006	*015	*024
	479	68 034	043	052	061	070	079	088	097	106	115
	480	68 124	133	142	151	160	169	178	187	196	205
	481	68 215	224	233	242	251	260	269	278	287	296
	482	68 305	314	323	332	341	350	359	368	377	386
	483	68 395	404	413	422	431	440	449	458	467	476
	484	68 485	494	502	511	520	529	538	547	556	565
	485	68 574	583	592	601	610	619	628	637	646	655
8 1 0.8 2 1.6 3 2.4 4 3.2 5 4.0 6 4.8 7 5.6 8 6.4 9 7.2	486	68 664	673	681	690	699	708	717	726	735	744
	487	68 753	762	771	780	789	797	806	815	824	833
	488	68 842	851	860	869	878	886	895	904	913	922
	489	68 931	940	949	958	966	975	984	993	*002	*011
	490	69 020	028	037	046	055	064	073	082	090	099
	491	69 108	117	126	135	144	152	161	170	179	188
	492	69 197	205	214	223	232	241	249	258	267	276
	493	69 285	294	302	311	320	329	338	346	355	364
	494	69 373	381	390	399	408	417	425	434	443	452
	495	69 461	469	478	487	496	504	513	522	531	539
	496	69 548	557	566	574	583	592	601	609	618	627
	497	69 636	644	653	662	671	679	688	697	705	714
	498	69 723	732	740	749	758	767	775	784	793	801
	499	69 810	819	827	836	845	854	862	871	880	888
	500	69 897	906	914	923	932	940	949	958	966	975
Prop. Parts	**N**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N	0	1	2	3	4	5	6	7	8	9	Prop. Parts
500	69 897	906	914	923	932	940	949	958	966	975	
501	69 984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70 070	079	088	096	105	114	122	131	140	148	
503	70 157	165	174	183	191	200	209	217	226	234	
504	70 243	252	260	269	278	286	295	303	312	321	
505	70 329	338	346	355	364	372	381	389	398	406	
506	70 415	424	432	441	449	458	467	475	484	492	
507	70 501	509	518	526	535	544	552	561	569	578	**9** 1 | 0.9 2 | 1.8 3 | 2.7 4 | 3.6 5 | 4.5 6 | 5.4 7 | 6.3 8 | 7.2 9 | 8.1
508	70 586	595	603	612	621	629	638	646	655	663	
509	70 672	680	689	697	706	714	723	731	740	749	
510	70 757	766	774	783	791	800	808	817	825	834	
511	70 842	851	859	868	876	885	893	902	910	919	
512	70 927	935	944	952	961	969	978	986	995	*003	
513	71 012	020	029	037	046	054	063	071	079	088	
514	71 096	105	113	122	130	139	147	155	164	172	
515	71 181	189	198	206	214	223	231	240	248	257	
516	71 265	273	282	290	299	307	315	324	332	341	
517	71 349	357	366	374	383	391	399	408	416	425	
518	71 433	441	450	458	466	475	483	492	500	508	
519	71 517	525	533	542	550	559	567	575	584	592	
520	71 600	609	617	625	634	642	650	659	667	675	
521	71 684	692	700	709	717	725	734	742	750	759	
522	71 767	775	784	792	800	809	817	825	834	842	**8** 1 | 0.8 2 | 1.6 3 | 2.4 3 | 3.2 5 | 4.0 6 | 4.8 7 | 5.6 8 | 6.4 9 | 7.2
523	71 850	858	867	875	883	892	900	908	917	925	
524	71 933	941	950	958	966	975	983	991	999	*008	
525	72 016	024	032	041	049	057	066	074	082	090	
526	72 099	107	115	123	132	140	148	156	165	173	
527	72 181	189	198	206	214	222	230	239	247	255	
528	72 263	272	280	288	296	304	313	321	329	337	
529	72 346	354	362	370	378	387	395	403	411	419	
530	72 428	436	444	452	460	469	477	485	493	501	
531	72 509	518	526	534	542	550	558	567	575	583	
532	72 591	599	607	616	624	632	640	648	656	665	
533	72 673	681	689	697	705	713	722	730	738	746	
534	72 754	762	770	779	787	795	803	811	819	827	
535	72 835	843	852	860	868	876	884	892	900	908	
536	72 916	925	933	941	949	957	965	973	981	989	**7** 1 | 0.7 2 | 1.4 3 | 2.1 4 | 2.8 5 | 3.5 6 | 4.2 7 | 4.9 8 | 5.6 9 | 6.3
537	72 997	*006	*014	*022	*030	*038	*046	*054	*062	*070	
538	73 078	086	094	102	111	119	127	135	143	151	
539	73 159	167	175	183	191	199	207	215	223	231	
540	73 239	247	255	263	272	280	288	296	304	312	
541	73 320	328	336	344	352	360	368	376	384	392	
542	73 400	408	416	424	432	440	448	456	464	472	
543	73 480	488	496	504	512	520	528	536	544	552	
544	73 560	568	576	584	592	600	608	616	624	632	
545	73 640	648	656	664	672	679	687	695	703	711	
546	73 719	727	735	743	751	759	767	775	783	791	
547	73 799	807	815	823	830	838	846	854	862	870	
548	73 878	886	894	902	910	918	926	933	941	949	
549	73 957	965	973	981	989	997	*005	*013	*020	*028	
550	74 036	044	052	060	068	076	084	092	099	107	
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**6**	**Prop. Parts**

X. 550—LOGARITHMS OF NUMBERS—600

Prop. Parts	N	0	1	2	3	4	5	6	7	8	9
	550	74 036	044	052	060	068	076	084	092	099	107
	551	74 115	123	131	139	147	155	162	170	178	186
	552	74 194	202	210	218	225	233	241	249	257	265
	553	74 273	280	288	296	304	312	320	327	335	343
	554	74 351	359	367	374	382	390	398	406	414	421
	555	74 429	437	445	453	461	468	476	484	492	500
	556	74 507	515	523	531	539	547	554	562	570	578
	557	74 586	593	601	609	617	624	632	640	648	656
	558	74 663	671	679	687	695	702	710	718	726	733
	559	74 741	749	757	764	772	780	788	796	803	811
	560	74 819	827	834	842	850	858	865	873	881	889
	561	74 896	904	912	920	927	935	943	950	958	966
	562	74 974	981	989	997	*005	*012	*020	*028	*035	*043
	563	75 051	059	066	074	082	089	097	105	113	120
	564	75 128	136	143	151	159	166	174	182	189	197
	565	75 205	213	220	228	236	243	251	259	266	274
	566	75 282	289	297	305	312	320	328	335	343	351
	567	75 358	366	374	381	389	397	404	412	420	427
	568	75 435	442	450	458	465	473	481	488	496	504
	569	75 511	519	526	534	542	549	557	565	572	580
	570	75 587	595	603	610	618	626	633	641	648	656
	571	75 664	671	679	686	694	702	709	717	724	732
	572	75 740	747	755	762	770	778	785	793	800	808
	573	75 815	823	831	838	846	853	861	868	876	884
	574	75 891	899	906	914	921	929	937	944	952	959
	575	75 967	974	982	989	997	*005	*012	*020	*027	*035
	576	76 042	050	057	065	072	080	087	095	103	110
	577	76 118	125	133	140	148	155	163	170	178	185
	578	76 193	200	208	215	223	230	238	245	253	260
	579	76 268	275	283	290	298	305	313	320	328	335
	580	76 343	350	358	365	373	380	388	395	403	410
	581	76 418	425	433	440	448	455	462	470	477	485
	582	76 492	500	507	515	522	530	537	545	552	559
	583	76 567	574	582	589	597	604	612	619	626	634
	584	76 641	649	656	664	671	678	686	693	701	708
	585	76 716	723	730	738	745	753	760	768	775	782
	586	76 790	797	805	812	819	827	834	842	849	856
	587	76 864	871	879	886	893	901	908	916	923	930
	588	76 938	945	953	960	967	975	982	989	997	*004
	589	77 012	019	026	034	041	048	056	063	070	078
	590	77 085	093	100	107	115	122	129	137	144	151
	591	77 159	166	173	181	188	195	203	210	217	225
	592	77 232	240	247	254	262	269	276	283	291	298
	593	77 305	313	320	327	335	342	349	357	364	371
	594	77 379	386	393	401	408	415	422	430	437	444
	595	77 452	459	466	474	481	488	495	503	510	517
	596	77 525	532	539	546	554	561	568	576	583	590
	597	77 597	605	612	619	627	634	641	648	656	663
	598	77 670	677	685	692	699	706	714	721	728	735
	599	77 743	750	757	764	772	779	786	793	801	808
	600	77 815	822	830	837	844	851	859	866	873	880
Prop. Parts	N	0	1	2	3	4	5	6	7	8	9

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

N	0	1	2	3	4	5	6	7	8	9	Prop. Parts
600	77 815	822	830	837	844	851	859	866	873	880	
601	77 887	895	902	909	916	924	931	938	945	952	
602	77 960	967	974	981	988	996	*003	*010	*017	*025	
603	78 032	039	046	053	061	068	075	082	089	097	
604	78 104	111	118	125	132	140	147	154	161	168	
605	78 176	183	190	197	204	211	219	226	233	240	
606	78 247	254	262	269	276	283	290	297	305	312	
607	78 319	326	333	340	347	355	362	369	376	383	8 1 0.8 2 1.6 3 2.4 4 3.2 5 4.0 6 4.8 7 5.6 8 6.4 9 7.2
608	78 390	398	405	412	419	426	433	440	447	455	
609	78 462	469	476	483	490	497	504	512	519	526	
610	78 533	540	547	554	561	569	576	583	590	597	
611	78 604	611	618	625	633	640	647	654	661	668	
612	78 675	682	689	696	704	711	718	725	732	739	
613	78 746	753	760	767	774	781	789	796	803	810	
614	78 817	824	831	838	845	852	859	866	873	880	
615	78 888	895	902	909	916	923	930	937	944	951	
616	78 958	965	972	979	986	993	*000	*007	*014	*021	
617	79 029	036	043	050	057	064	071	078	085	092	
618	79 099	106	113	120	127	134	141	148	155	162	
619	79 169	176	183	190	197	204	211	218	225	232	
620	79 239	246	253	260	267	274	281	288	295	302	
621	79 309	316	323	330	337	344	351	358	365	372	7 1 0.7 2 1.4 3 2.1 4 2.8 5 3.5 6 4.2 7 4.9 8 5.6 9 6.3
622	79 379	386	393	400	407	414	421	428	435	442	
623	79 449	456	463	470	477	484	491	498	505	511	
624	79 518	525	532	539	546	553	560	567	574	581	
625	79 588	595	602	609	616	623	630	637	644	650	
626	79 657	664	671	678	685	692	699	706	713	720	
627	79 727	734	741	748	754	761	768	775	782	789	
628	79 796	803	810	817	824	831	837	844	851	858	
629	79 865	872	879	886	893	900	906	913	920	927	
630	79 934	941	948	955	962	969	975	982	989	996	
631	80 003	010	017	024	030	037	044	051	058	065	
632	80 072	079	085	092	099	106	113	120	127	134	
633	80 140	147	154	161	168	175	182	188	195	202	
634	80 209	216	223	229	236	243	250	257	264	271	
635	80 277	284	291	298	305	312	318	325	332	339	
636	80 346	353	359	366	373	380	387	393	400	407	6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4
637	80 414	421	428	434	441	448	455	462	468	475	
638	80 482	489	496	502	509	516	523	530	536	543	
639	80 550	557	564	570	577	584	591	598	604	611	
640	80 618	625	632	638	645	652	659	665	672	679	
641	80 686	693	699	706	713	720	726	733	740	747	
642	80 754	760	767	774	781	787	794	801	808	814	
643	80 821	828	835	841	848	855	862	868	875	882	
644	80 889	895	902	909	916	922	929	936	943	949	
645	80 956	963	969	976	983	990	996	*003	*010	*017	
646	81 023	030	037	043	050	057	064	070	077	084	
647	81 090	097	104	111	117	124	131	137	144	151	
648	81 158	164	171	178	184	191	198	204	211	218	
649	81 224	231	238	245	251	258	265	271	278	285	
650	81 291	298	305	311	318	325	331	338	345	351	
N	0	1	2	3	4	5	6	7	8	9	Prop. Parts

Prop. Parts	N	0	1	2	3	4	5	6	7	8	9
	650	81 291	298	305	311	318	325	331	338	345	351
	651	81 358	365	371	378	385	391	398	405	411	418
	652	81 425	431	438	445	451	458	465	471	478	485
	653	81 491	498	505	511	518	525	531	538	544	551
	654	81 558	564	571	578	584	591	598	604	611	617
	655	81 624	631	637	644	651	657	664	671	677	684
	656	81 690	697	704	710	717	723	730	737	743	750
	657	81 757	763	770	776	783	790	796	803	809	816
	658	81 823	829	836	842	849	856	862	869	875	882
	659	81 889	895	902	908	915	921	928	935	941	948
	660	81 954	961	968	974	981	987	994	*000	*007	*014
	661	82 020	027	033	040	046	053	060	066	073	079
7 1 0.7 2 1.4 3 2.1 4 2.8 5 3.5 6 4.2 7 4.9 8 5.6 9 6.3	662	82 086	092	099	105	112	119	125	132	138	145
	663	82 151	158	164	171	178	184	191	197	204	210
	664	82 217	223	230	236	243	249	256	263	269	276
	665	82 282	289	295	302	308	315	321	328	334	341
	666	82 347	354	360	367	373	380	387	393	400	406
	667	82 413	419	426	432	439	445	452	458	465	471
	668	82 478	484	491	497	504	510	517	523	530	536
	669	82 543	549	556	562	569	575	582	588	595	601
	670	82 607	614	620	627	633	640	646	653	659	666
	671	82 672	679	685	692	698	705	711	718	724	730
	672	82 737	743	750	756	763	769	776	782	789	795
	673	82 802	808	814	821	827	834	840	847	853	860
	674	82 866	872	879	885	892	898	905	911	918	924
	675	82 930	937	943	950	956	963	969	975	982	988
	676	82 995	*001	*008	*014	*020	*027	*033	*040	*046	*052
	677	83 059	065	072	078	085	091	097	104	110	117
	678	83 123	129	136	142	149	155	161	168	174	181
	679	83 187	193	200	206	213	219	225	232	238	245
	680	83 251	257	264	270	276	283	289	296	302	308
6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4	681	83 315	321	327	334	340	347	353	359	366	372
	682	83 378	385	391	398	404	410	417	423	429	436
	683	83 442	448	455	461	467	474	480	487	493	499
	684	83 506	512	518	525	531	537	544	550	556	563
	685	83 569	575	582	588	594	601	607	613	620	626
	686	83 632	639	645	651	658	664	670	677	683	689
	687	83 696	702	708	715	721	727	734	740	746	753
	688	83 759	765	771	778	784	790	797	803	809	816
	689	83 822	828	835	841	847	853	860	866	872	879
	690	83 885	891	897	904	910	916	923	929	935	942
	691	83 948	954	960	967	973	979	985	992	998	*004
	692	84 011	017	023	029	036	042	048	055	061	067
	693	84 073	080	086	092	098	105	111	117	123	130
	694	84 136	142	148	155	161	167	173	180	186	192
	695	84 198	205	211	217	223	230	236	242	248	255
	696	84 261	267	273	280	286	292	298	305	311	317
	697	84 323	330	336	342	348	354	361	367	373	379
	698	84 386	392	398	404	410	417	423	429	435	442
	699	84 448	454	460	466	473	479	485	491	497	504
	700	84 510	516	522	528	535	541	547	553	559	566
Prop. Parts	**N**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N	0	1	2	3	4	5	6	7	8	9	Prop. Parts
700	84 510	516	522	528	535	541	547	553	559	566	
701	84 572	578	584	590	597	603	609	615	621	628	
702	84 634	640	646	652	658	665	671	677	683	689	
703	84 696	702	708	714	720	726	733	739	745	751	
704	84 757	763	770	776	782	788	794	800	807	813	
705	84 819	825	831	837	844	850	856	862	868	874	
706	84 880	887	893	899	905	911	917	924	930	936	
707	84 942	948	954	960	967	973	979	985	991	997	
708	85 003	009	016	022	028	034	040	046	052	058	
709	85 065	071	077	083	089	095	101	107	114	120	
710	85 126	132	138	144	150	156	163	169	175	181	
711	85 187	193	199	205	211	217	224	230	236	242	
712	85 248	254	260	266	272	278	285	291	297	303	
713	85 309	315	321	327	333	339	345	352	358	364	
714	85 370	376	382	388	394	400	406	412	418	425	
715	85 431	437	443	449	455	461	467	473	479	485	
716	85 491	497	503	509	516	522	528	534	540	546	
717	85 552	558	564	570	576	582	588	594	600	606	
718	85 612	618	625	631	637	643	649	655	661	667	
719	85 673	679	685	691	697	703	709	715	721	727	
720	85 733	739	745	751	757	763	769	775	781	788	
721	85 794	800	806	812	818	824	830	836	842	848	
722	85 854	860	866	872	878	884	890	896	902	908	
723	85 914	920	926	932	938	944	950	956	962	968	
724	85 974	980	986	992	998	*004	*010	*016	*022	*028	
725	86 034	040	046	052	058	064	070	076	082	088	
726	86 094	100	106	112	118	124	130	136	141	147	
727	86 153	159	165	171	177	183	189	195	201	207	
728	86 213	219	225	231	237	243	249	255	261	267	
729	86 273	279	285	291	297	303	308	314	320	326	
730	86 332	338	344	350	356	362	368	374	380	386	
731	86 392	398	404	410	415	421	427	433	439	445	
732	86 451	457	463	469	475	481	487	493	499	504	
733	86 510	516	522	528	534	540	546	552	558	564	
734	86 570	576	581	587	593	599	605	611	617	623	
735	86 629	635	641	646	652	658	664	670	676	682	
736	86 688	694	700	705	711	717	723	729	735	741	
737	86 747	753	759	764	770	776	782	788	794	800	
738	86 806	812	817	823	829	835	841	847	853	859	
739	86 864	870	876	882	888	894	900	906	911	917	
740	86 923	929	935	941	947	953	958	964	970	976	
741	86 982	988	994	999	*005	*011	*017	*023	*029	*035	
742	87 040	046	052	058	064	070	075	081	087	093	
743	87 099	105	111	116	122	128	134	140	146	151	
744	87 157	163	169	175	181	186	192	198	204	210	
745	87 216	221	227	233	239	245	251	256	262	268	
746	87 274	280	286	291	297	303	309	315	320	326	
747	87 332	338	344	349	355	361	367	373	379	384	
748	87 390	396	402	408	413	419	425	431	437	442	
749	87 448	454	460	466	471	477	483	489	495	500	
750	87 506	512	518	523	529	535	541	547	552	558	
N	0	1	2	3	4	5	6	7	8	9	Prop. Parts

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Prop. Parts	N	0	1	2	3	4	5	6	7	8	9
	750	87 506	512	518	523	529	535	541	547	552	558
	751	87 564	570	576	581	587	593	599	604	610	616
	752	87 622	628	633	639	645	651	656	662	668	674
	753	87 679	685	691	697	703	708	714	720	726	731
	754	87 737	743	749	754	760	766	772	777	783	789
	755	87 795	800	806	812	818	823	829	835	841	846
	756	87 852	858	864	869	875	881	887	892	898	904
	757	87 910	915	921	927	933	938	944	950	955	961
	758	87 967	973	978	984	990	996	*001	*007	*013	*018
	759	88 024	030	036	041	047	053	058	064	070	076
	760	88 081	087	093	098	104	110	116	121	127	133
	761	88 138	144	150	156	161	167	173	178	184	190
6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4	762	88 195	201	207	213	218	224	230	235	241	247
	763	88 252	258	264	270	275	281	287	292	298	304
	764	88 309	315	321	326	332	338	343	349	355	360
	765	88 366	372	377	383	389	395	400	406	412	417
	766	88 423	429	434	440	446	451	457	463	468	474
	767	88 480	485	491	497	502	508	513	519	525	530
	768	88 536	542	547	553	559	564	570	576	581	587
	769	88 593	598	604	610	615	621	627	632	638	643
	770	88 649	655	660	666	672	677	683	689	694	700
	771	88 705	711	717	722	728	734	739	745	750	756
	772	88 762	767	773	779	784	790	795	801	807	812
	773	88 818	824	829	835	840	846	852	857	863	868
	774	88 874	880	885	891	897	902	908	913	919	925
	775	88 930	936	941	947	953	958	964	969	975	981
	776	88 986	992	997	*003	*009	*014	*020	*025	*031	*037
	777	89 042	048	053	059	064	070	076	081	087	092
	778	89 098	104	109	115	120	126	131	137	143	148
	779	89 154	159	165	170	176	182	187	193	198	204
	780	89 209	215	221	226	232	237	243	248	254	260
5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5	781	89 265	271	276	282	287	293	298	304	310	315
	782	89 321	326	332	337	343	348	354	360	365	371
	783	89 376	382	387	393	398	404	409	415	421	426
	784	89 432	437	443	448	454	459	465	470	476	481
	785	89 487	492	498	504	509	515	520	526	531	537
	786	89 542	548	553	559	564	570	575	581	586	592
	787	89 597	603	609	614	620	625	631	636	642	647
	788	89 653	658	664	669	675	680	686	691	697	702
	789	89 708	713	719	724	730	735	741	746	752	757
	790	89 763	768	774	779	785	790	796	801	807	812
	791	89 818	823	829	834	840	845	851	856	862	867
	792	89 873	878	883	889	894	900	905	911	916	922
	793	89 927	933	938	944	949	955	960	966	971	977
	794	89 982	988	993	998	*004	*009	*015	*020	*026	*031
	795	90 037	042	048	053	059	064	069	075	080	086
	796	90 091	097	102	108	113	119	124	129	135	140
	797	90 146	151	157	162	168	173	179	184	189	195
	798	90 200	206	211	217	222	227	233	238	244	249
	799	90 255	260	266	271	276	282	287	293	298	304
	800	90 309	314	320	325	331	336	342	347	352	358
Prop. Parts	**N**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N	0	1	2	3	4	5	6	7	8	9
800	90 309	314	320	325	331	336	342	347	352	358
801	90 363	369	374	380	385	390	396	401	407	412
802	90 417	423	428	434	439	445	450	455	461	466
803	90 472	477	482	488	493	499	504	509	515	520
804	90 526	531	536	542	547	553	558	563	569	574
805	90 580	585	590	596	601	607	612	617	623	628
806	90 634	639	644	650	655	660	666	671	677	682
807	90 687	693	698	703	709	714	720	725	730	736
808	90 741	747	752	757	763	768	773	779	784	789
809	90 795	800	806	811	816	822	827	832	838	843
810	90 849	854	859	865	870	875	881	886	891	897
811	90 902	907	913	918	924	929	934	940	945	950
812	90 956	961	966	972	977	982	988	993	998	*004
813	91 009	014	020	025	030	036	041	046	052	057
814	91 062	068	073	078	084	089	094	100	105	110
815	91 116	121	126	132	137	142	148	153	158	164
816	91 169	174	180	185	190	196	201	206	212	217
817	91 222	228	233	238	243	249	254	259	265	270
818	91 275	281	286	291	297	302	307	312	318	323
819	91 328	334	339	344	350	355	360	365	371	376
820	91 381	387	392	397	403	408	413	418	424	429
821	91 434	440	445	450	455	461	466	471	477	482
822	91 487	492	498	503	508	514	519	524	529	535
823	91 540	545	551	556	561	566	572	577	582	587
824	91 593	598	603	609	614	619	624	630	635	640
825	91 645	651	656	661	666	672	677	682	687	693
826	91 698	703	709	714	719	724	730	735	740	745
827	91 751	756	761	766	772	777	782	787	793	798
828	91 803	808	814	819	824	829	834	840	845	850
829	91 855	861	866	871	876	882	887	892	897	903
830	91 908	913	918	924	929	934	939	944	950	955
831	91 960	965	971	976	981	986	991	997	*002	*007
832	92 012	018	023	028	033	038	044	049	054	059
833	92 065	070	075	080	085	091	096	101	106	111
834	92 117	122	127	132	137	143	148	153	158	163
835	92 169	174	179	184	189	195	200	205	210	215
836	92 221	226	231	236	241	247	252	257	262	267
837	92 273	278	283	288	293	298	304	309	314	319
838	92 324	330	335	340	345	350	355	361	366	371
839	92 376	381	387	392	397	402	407	412	418	423
840	92 428	433	438	443	449	454	459	464	469	474
841	92 480	485	490	495	500	505	511	516	521	526
842	92 531	536	542	547	552	557	562	567	572	578
843	92 583	588	593	598	603	609	614	619	624	629
844	92 634	639	645	650	655	660	665	670	675	681
845	92 686	691	696	701	706	711	716	722	727	732
846	92 737	742	747	752	758	763	768	773	778	783
847	92 788	793	799	804	809	814	819	824	829	834
848	92 840	845	850	855	860	865	870	875	881	886
849	92 891	896	901	906	911	916	921	927	932	937
850	92 942	947	952	957	962	967	973	978	983	988
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Parts

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Prop. Parts

Prop. Parts	N	0	1	2	3	4	5	6	7	8	9
	850	92 942	947	952	957	962	967	973	978	983	988
	851	92 993	998	*003	*008	*013	*018	*024	*029	*034	*039
	852	93 044	049	054	059	064	069	075	080	085	090
	853	93 095	100	105	110	115	120	125	131	136	141
	854	93 146	151	156	161	166	171	176	181	186	192
	855	93 197	202	207	212	217	222	227	232	237	242
	856	93 247	252	258	263	268	273	278	283	288	293
6	857	93 298	303	308	313	318	323	328	334	339	344
1 0.6	858	93 349	354	359	364	369	374	379	384	389	394
2 1.2	859	93 399	404	409	414	420	425	430	435	440	445
3 1.8	**860**	93 450	455	460	465	470	475	480	485	490	495
4 2.4	861	93 500	505	510	515	520	526	531	536	541	546
5 3.0	862	93 551	556	561	566	571	576	581	586	591	596
6 3.6	863	93 601	606	611	616	621	626	631	636	641	646
7 4.2	864	93 651	656	661	666	671	676	682	687	692	697
8 4.8	865	93 702	707	712	717	722	727	732	737	742	747
9 5.4	866	93 752	757	762	767	772	777	782	787	792	797
	867	93 802	807	812	817	822	827	832	837	842	847
	868	93 852	857	862	867	872	877	882	887	892	897
	869	93 902	907	912	917	922	927	932	937	942	947
	870	93 952	957	962	967	972	977	982	987	992	997
	871	94 002	007	012	017	022	027	032	037	042	047
5	872	94 052	057	062	067	072	077	082	086	091	096
1 0.5	873	94 101	106	111	116	121	126	131	136	141	146
2 1.0	874	94 151	156	161	166	171	176	181	186	191	196
3 1.5	875	94 201	206	211	216	221	226	231	236	240	245
4 2.0	876	94 250	255	260	265	270	275	280	285	290	295
5 2.5	877	94 300	305	310	315	320	325	330	335	340	345
6 3.0	878	94 349	354	359	364	369	374	379	384	389	394
7 3.5	879	94 399	404	409	414	419	424	429	433	438	443
8 4.0	**880**	94 448	453	458	463	468	473	478	483	488	493
9 4.5	881	94 498	503	507	512	517	522	527	532	537	542
	882	94 547	552	557	562	567	571	576	581	586	591
	883	94 596	601	606	611	616	621	626	630	635	640
	884	94 645	650	655	660	665	670	675	680	685	689
	885	94 694	699	704	709	714	719	724	729	734	738
4	886	94 743	748	753	758	763	768	773	778	783	787
1 0.4	887	94 792	797	802	807	812	817	822	827	832	836
2 0.8	888	94 841	846	851	856	861	866	871	876	880	885
3 1.2	889	94 890	895	900	905	910	915	919	924	929	934
4 1.6	**890**	94 939	944	949	954	959	963	968	973	978	983
5 2.0	891	94 988	993	998	*002	*007	*012	*017	*022	*027	*032
6 2.4	892	95 036	041	046	051	056	061	066	071	075	080
7 2.8	893	95 085	090	095	100	105	109	114	119	124	129
8 3.2	894	95 134	139	143	148	153	158	163	168	173	177
9 3.6	895	95 182	187	192	197	202	207	211	216	221	226
	896	95 231	236	240	245	250	255	260	265	270	274
	897	95 279	284	289	294	299	303	308	313	318	323
	898	95 328	332	337	342	347	352	357	361	366	371
	899	95 376	381	386	390	395	400	405	410	415	419
	900	95 424	429	434	439	444	448	453	458	463	468
Prop. Parts	**N**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N	0	1	2	3	4	5	6	7	8	9	Prop. Parts
900	95 424	429	434	439	444	448	453	458	463	468	
901	95 472	477	482	487	492	497	501	506	511	516	
902	95 521	525	530	535	540	545	550	554	559	564	
903	95 569	574	578	583	588	593	598	602	607	612	
904	95 617	622	626	631	636	641	646	650	655	660	
905	95 665	670	674	679	684	689	694	698	703	708	
906	95 713	718	722	727	732	737	742	746	751	756	
907	95 761	766	770	775	780	785	789	794	799	804	
908	95 809	813	818	823	828	832	837	842	847	852	
909	95 856	861	866	871	875	880	885	890	895	899	
910	95 904	909	914	018	923	928	933	938	942	947	
911	95 952	957	961	966	971	976	980	985	990	995	
912	95 999	*004	*009	*014	*019	*023	*028	*033	*038	*042	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
913	96 047	052	057	061	066	071	076	080	085	090	
914	96 095	099	104	109	114	118	123	128	133	137	
915	96 142	147	152	156	161	166	171	175	180	185	
916	96 190	194	199	204	209	213	218	223	227	232	
917	96 237	242	246	251	256	261	265	270	275	280	
918	96 284	289	294	298	303	308	313	317	322	327	
919	96 332	336	341	346	350	355	360	365	369	374	
920	96 379	384	388	393	398	402	407	412	417	421	
921	96 426	431	435	440	445	450	454	459	464	468	
922	96 473	478	483	487	492	497	501	506	511	515	
923	96 520	525	530	534	539	544	548	553	558	562	
924	96 567	572	577	581	586	591	595	600	605	609	
925	96 614	619	624	628	633	638	642	647	652	656	
926	96 661	666	670	675	680	685	689	694	699	703	
927	96 708	713	717	722	727	731	736	741	745	750	
928	96 755	759	764	769	774	778	783	788	792	797	
929	96 802	806	811	816	820	825	830	834	839	844	
930	96 848	853	858	862	867	872	876	881	886	890	
931	96 895	900	904	909	914	918	923	928	932	937	
932	96 942	946	951	956	960	965	970	974	979	984	4 1 0.4 2 0.8 3 1.2 4 1.6 5 2.0 6 2.4 7 2.8 8 3.2 9 3.6
933	96 988	993	997	*002	*007	*011	*016	*021	*025	*030	
934	97 035	039	044	049	053	058	063	067	072	077	
935	97 081	086	090	095	100	104	109	114	118	123	
936	97 128	132	137	142	146	151	155	160	165	169	
937	97 174	179	183	188	192	197	202	206	211	216	
938	97 220	225	230	234	239	243	248	253	257	262	
939	97 267	271	276	280	285	290	294	299	304	308	
940	97 313	317	322	327	331	336	340	345	350	354	
941	97 359	364	368	373	377	382	387	391	396	400	
942	97 405	410	414	419	424	428	433	437	442	447	
943	97 451	456	460	465	470	474	479	483	488	493	
944	97 497	502	506	511	516	520	525	529	534	539	
945	97 543	548	552	557	562	566	571	575	580	585	
946	97 589	594	598	603	607	612	617	621	626	630	
947	97 635	640	644	649	653	658	663	667	672	676	
948	97 681	685	690	695	699	704	708	713	717	722	
949	97 727	731	736	740	745	749	754	759	763	768	
950	97 772	777	782	786	791	795	800	804	809	813	
N	0	1	2	3	4	5	6	7	8	9	Prop. Parts

Prop. Parts	N	0	1	2	3	4	5	6	7	8	9
	950	97 772	777	782	786	791	795	800	804	809	813
	951	97 818	823	827	832	836	841	845	850	855	859
	952	97 864	868	873	877	882	886	891	896	900	905
	953	97 909	914	918	923	928	932	937	941	946	950
	954	97 955	959	964	968	973	978	982	987	991	996
	955	98 000	005	009	014	019	023	028	032	037	041
	956	98 046	050	055	059	064	068	073	078	082	087
	957	98 091	096	100	105	109	114	118	123	127	132
	958	98 137	141	146	150	155	159	164	168	173	177
	959	98 182	186	191	195	200	204	209	214	218	223
	960	98 227	232	236	241	245	250	254	259	263	268
	961	98 272	277	281	286	290	295	299	304	308	313
5	962	98 318	322	327	331	336	340	345	349	354	358
1 0.5	963	98 363	367	372	376	381	385	390	394	399	403
2 1.0 3 1.5	964	98 408	412	417	421	426	430	435	439	444	448
4 2.0	965	98 453	457	462	466	471	475	480	484	489	493
5 2.5 6 3.0	966	98 498	502	507	511	516	520	525	529	534	538
7 3.5	967	98 543	547	552	556	561	565	570	574	579	583
8 4.0	968	98 588	592	597	601	605	610	614	619	623	628
9 4.5	969	98 632	637	641	646	650	655	659	664	668	673
	970	98 677	682	686	691	695	700	704	709	713	717
	971	98 722	726	731	735	740	744	749	753	758	762
	972	98 767	771	776	780	784	789	793	798	802	807
	973	98 811	816	820	825	829	834	838	843	847	851
	974	98 856	860	865	869	874	878	883	887	892	896
	975	98 900	905	909	914	918	923	927	932	936	941
	976	98 945	949	954	958	963	967	972	976	981	985
	977	98 989	994	998	*003	*007	*012	*016	*021	*025	*029
	978	99 034	038	043	047	052	056	061	065	069	074
	979	99 078	083	087	092	096	100	105	109	114	118
	980	99 123	127	131	136	140	145	149	154	158	162
4	981	99 167	171	176	180	185	189	193	198	202	207
1 0.4	982	99 211	216	220	224	229	233	238	242	247	251
2 0.8	983	99 255	260	264	269	273	277	282	286	291	295
3 1.2 4 1.6	984	99 300	304	308	313	317	322	326	330	335	339
5 2.0	985	99 344	348	352	357	361	366	370	374	379	383
6 2.4	986	99 388	392	396	401	405	410	414	419	423	427
7 2.8 8 3.2	987	99 432	436	441	445	449	454	458	463	467	471
9 3.6	988	99 476	480	484	489	493	498	502	506	511	515
	989	99 520	524	528	533	537	542	546	550	555	559
	990	99 564	568	572	577	581	585	590	594	599	603
	991	99 607	612	616	621	625	629	634	638	642	647
	992	99 651	656	660	664	669	673	677	682	686	691
	993	99 695	699	704	708	712	717	721	726	730	734
	994	99 739	743	747	752	756	760	765	769	774	778
	995	99 782	787	791	795	800	804	808	813	817	822
	996	99 826	830	835	839	843	848	852	856	861	865
	997	99 870	874	878	883	887	891	896	900	904	909
	998	99 913	917	922	926	930	935	939	944	948	952
	999	99 957	961	965	970	974	978	983	987	991	996
	1000	00 000	004	009	013	017	022	026	030	035	039
Prop. Parts	**N**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**